# SEQUENTIAL AND SIMULTANEOUS SLAR BLOCK ADJUSTMENT[1]

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## ABSTRACT

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For Side Looking Airborne Radar (SLAR) mapping projects of extensive flat regions such as the Amazon basin, mosaicking procedures have been in use. In the particular case of Colombia's PRORADAM (Projecto Radargrammetrico del Amazonas), a simple sequential SLAR block adjustment produced the metric base for mosaicking. The present report analyzes the relative merits of the sequential versus three methods of simultaneous adjustment of a SLAR block. It is demonstrated that sequential block formation with spline functions, followed by external interpolative adjustment produces very good results. Simultaneous planimetric block adjustment with similarity transformations (ANBLOCK), affine transformations or transformations with spline functions cannot easily approach or surpass these results. These conclusions were obtained on the basis of controlled experiments with simulated SLAR imagery, implementing parameters of actual SLAR mapping projects.

## INTRODUCTION

The paper reports on an attempt to define an optimum method of radargrammetric point determination in a flat mapping area. It will show that sequential block formation followed by external adjustment of the block can produce accurate results not easily attainable with simultaneous block adjustment with the principles of ANBLOCK[2] or spline functions.

The problem of radargrammetric point determination with a block of overlapping Side Looking Airborne Radar (SLAR) images was proposed for the first time in the Colombian "Projecto Radargrammetrico del Amazonas (Proradam)". In that project severe time constraints dictated an intuitive choice of computational procedures, which had to be simple. Therefore sequential block formation and external adjustment were used (Leberl, 1975).

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[1] Paper presented at the Annual Convention of the American Society of Photogrammetry, Washington, D.C., March 1975.

[2] Simultaneous planimetric block adjustment using 4-parameter similarity transformations.

The analysis presented in this paper started out as an afterthought to the actual radargrammetric work for Proradam. It is based on simulated SLAR imagery. The following methods of computing a SLAR block adju ment were studied: sequential block formation with and without spline functions; simultaneous adjustment with 4-parameter transformations according to the photogrammetric ANBLOCK principle; with 5-parameter transformations; and with piecewise polynomial transformation.

After an outline of SLAR imagery deformations, these computation methods will be explained next, along with the simulated block of SLAR imagery. Then the results of the computations are analyzed. The main question addressed in the analysis will be whether any gain can be expec in radargrammetric point determination, when simple sequential methods of computation are replaced by a more involved simultaneous SLAR bloc adjustment. For conciseness the sensitivity of the methods to the various project parameters will not be treated; instead, the performance of differ computation methods is studied with one set of project parameters.

## SIDE LOOKING AIRBORNE RADAR IMAGERY DEFORMATIONS

An ideal strip of SLAR imagery of flat terrain flown along a meridian represents a transverse cylindrical equidistant projection, with the flight-line as the reference meridian. If no image deformations occurred, then radargrammetric point determination would be simply a transformation of each individual cylindrical projection into the desired map projection.

Unfortunately, however, SLAR images are usually deformed in spite of inertial navigation, gyroscopic stabilization of the antennae, electronic co pensation of detected errors of attitude and position of the sensor, and well controlled image formation aboard the survey plane and in the optic correlator.

What are the causes of errors in image geometry? Film transport and alignment aboard the airplane as well as in the correlator is erroneous: transport velocity is not perfectly uniform, and the film may "wander" in a direction transverse to the forward movement. Sensing of attitude a position of the sensor is imperfect: here, however, attitude errors do not propagate into the geometry of a synthetic aperture radar image.The orientation of the (synthetic) radar beam is perpendicular to the flightlin Errors of position, however, may propagate fully into the geometry of a SLAR image: a flightline will not be perfectly parallel to a meridian; instead errors of inertial navigation will cause a long term periodical deviation of the flightline from the meridian, and also cause similar long term variations of aircraft speed (the deviations have a so-called Schuler-frequency). In addition, flight attitude can vary without being properly compensated for in the formation of the image. And finally, of course, t "flat" mapping area might exhibit some relief.

Which of these error causes are dominant? Experiences with SLAR mapping projects in the past (Van Roessel and De Godoy, 1974; Leberl 1975) have shown that the Schuler-frequency of inertial navigation produce image errors of the order of magnitude of several kilometers, or millimeters at the scale of imagery. This is much more serious than slight relief: a height difference of 160 m, for example, would produce a maximal parallax difference of 0.2 mm at scale 1:400,000. This is also the order of magnitude of other image deformations, as e.g., due to erroneous film transport or alignment.

In conclusion it is therefore justified to model a SLAR block adjustment to the specific effect of the Schuler-frequency of inertial navigation.

Figs. 1a, b and c illustrate with the help of a grid, how the error of inertial navigation can deform a SLAR image. Defining an $x$, $y$ image coordinate system as shown in Fig. 1c one can plot image errors $\Delta x$ and $\Delta y$ as functions of $x$. This is illustrated in Fig. 2.

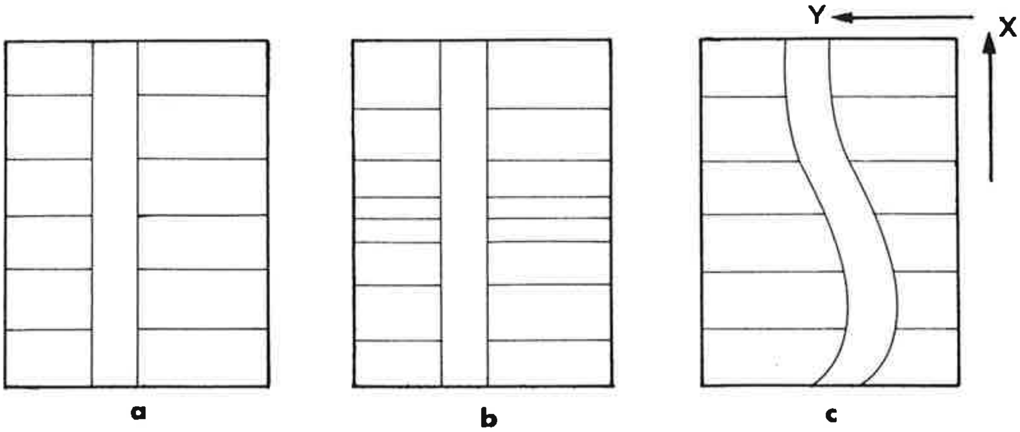


Fig. 1. Effect of error of inertial navigation on SLAR image of a grid. $X$-axis coincides with flight direction. In case $c$, grid lines are not perpendicular, if antenna is gyroscopically stabilized and radar has real aperture, but would remain perpendicular with synthetic aperture.

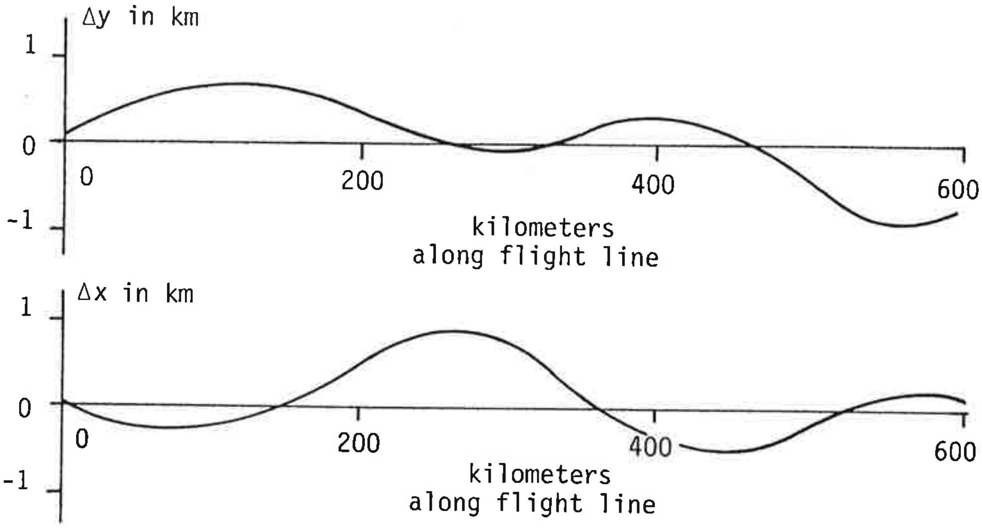


Fig. 2. Deviations of simulated flight no. 1 from a straight line at constant speed.

For the purpose of the analysis, a set of twenty SLAR images with 60% sidelap was simulated, showing $\Delta x$ and $\Delta y$ deformations of the type presented in Fig. 2. The length of each strip was chosen to be 600 km. The particular assumptions for the flight configuration are given in Fig. 3, and reflect the actual flight planning used in Colombia's "Proradam" and other SLAR projects.

Two sets of SLAR images were simulated: one only showing the $\Delta x$, $\Delta y$ errors due to inertial navigation; and a second one showing these same errors and in addition effects of random noise and of variations of the flight height. Standard deviation of noise was assumed to be ± 80 m (± 0.2 mm at scale 1:400,000).

These simulated sets of SLAR images neglect a number of secondary image errors. Consequently, the absolute quantitative results of the analysis will be somewhat optimistic. But of interest is a relative comparison of computational procedures. For this purpose the level of refinement of the simulation is considered sufficient.

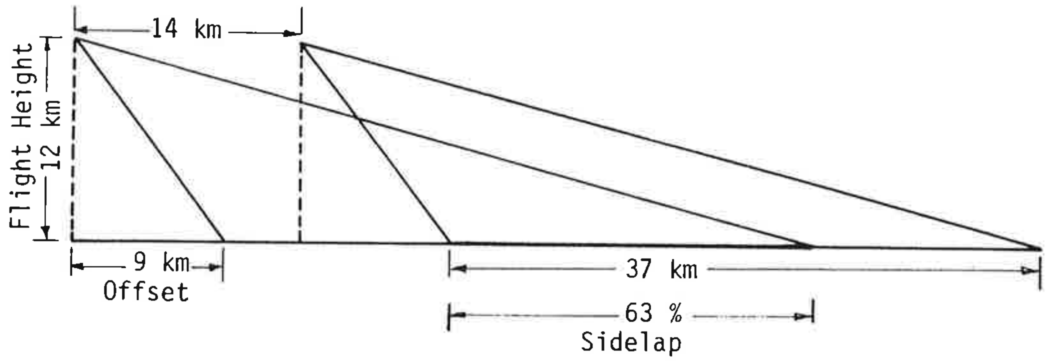


Fig. 3. Flight configuration used in simulation, and in PRORADAM.

## PLANIMETRIC SLAR BLOCK ADJUSTMENT METHODS

### *Sequential methods*

Sequential formation of a block with subsequent interpolative adjustment ("external block adjustment") is a simple, fast and inexpensive method of point determination, requiring only a limited effort of computer programming. For these reasons, this method was the one most suitable for use in "Proradam."

Starting from an initial strip of imagery, the adjacent strip is transformed into the system of the previous one using the common tiepoints in the overlapping area. Since this transformation will generally be overdetermined, discrepancies will be left between the two sets of tiepoints after transformation. In order to obtain a unique pair of $x$, $y$ block coordinates of tiepoints, the arithmetic mean of the two sets of coordinates of each tiepoint is chosen.

Two alternatives for sequential block formation were available: one is based on similarity transformation of the entire strips into the block system; the other uses the $\Delta x$, $\Delta y$ residuals left after the similarity transformation to compute a piecewise third-order polynomial of the form:

$$\begin{aligned} \Delta x &= a_{i0} + a_{i1}(x-x_i) + a_{i2}(x-x_i)^2 + a_{i3}(x-x_i)^3 \\ \Delta y &= b_{i0} + b_{i1}(x-x_i) + b_{i2}(x-x_i)^2 + b_{i3}(x-x_i)^3 \end{aligned} \quad (1)$$

where:

$$x_i \leqslant x \leqslant x_{i+1}$$

With the piecewise polynomials, discrepancies between adjacent strips can be reduced considerably as compared to the similarity transformation. However, since the polynomials are functions of only $x$, and independent of $y$, they will only be applicable if discrepancies $\Delta x$, $\Delta y$ are indeed independent of $y$. Inertial Schuler-frequency does not result in image errors as a function of $y$ (Fig. 1). But secondary defects of image geometry, such as due to an erroneous flight height, can produce a $\Delta x$, $\Delta y$ as a function of $y$. These defects, however, are an entire order of magnitude smaller than those due to the Schuler-frequency.

The formation of the block is followed by external adjustment. The block is transformed into the system of ground control points and an interpolation and filtering algorithm is applied to correct the radargrammetric points using the known block deformations in control points. Essentially, the interpolation algorithm used is linear prediction (Kraus and Mikhail, 1972).

### *Simultaneous methods*

*Similarity transformations.* Three algorithms were programmed for the analysis. The well-known ANBLOCK method of photogrammetric planimetric block adjustment simultaneously transforms each SLAR strip, or part thereof, according to:

$$\begin{aligned} X &= ax + by + c \\ Y &= -bx + ay + d \end{aligned} \quad (2)$$

Consequently, this ANBLOCK method is the simultaneous equivalent of the above sequential block formation using similarity transformations.

*Transformation with differential scale.* Scales in $x$ and $y$ are independent of each other in SLAR. Although they should be identical, there are numerous causes resulting in scale differences in along-track and across-track directions. Therefore it seems logical to expand the traditional

ANBLOCK concept and not to use a strict similarity transformation sucl as eq. (2), but to allow for differential scale, according to:

$$X = \lambda\cos\alpha(x)+\lambda\sin\alpha(y)+c$$
$$Y = -(\lambda+\Delta\lambda)\sin\alpha(x)+(\lambda+\Delta\lambda)\cos\alpha(y)+d \quad (3)$$

Here, $\lambda$ is scale, $\alpha$ the rotation, and $\Delta\lambda$ a differential scale. If $(x,y)$ and $(X,Y)$ coordinate systems are nearly parallel, and $\alpha$ is thus small, express (3) simplifies to:

$$X = ax + by + c$$
$$Y = -bx + ay + d + ey \quad (4)$$

in which the term $ey$ thus allows for a differential scale. A simultaneous block adjustment with transformation (4) was programmed and is called AFBLOCK.

*Transformation with piecewise polynomials.* An algorithm was developed, in which the Schuler-frequencies are modeled as piecewise polynomials, similar to the approach in formula (1). The transformation parameters a not the coefficients of a polynomial, but rather function values of the continuous $\Delta x$, $\Delta y$ image coordinate errors. Again, it is assumed that $\Delta x$ $\Delta y$ are functions of $x$ only. With reference to Fig. 4, these functions ar described by function values $e_i$ and $f_i$, such that:

$$X = ax + by + \overbrace{e_i\left(1-\frac{x-x_i}{x_{i+1}-x_i}\right) + e_{i+1}\frac{x-x_i}{x_{i+1}-x_i}}^{\Delta x}$$
$$Y = -by + ax + \underbrace{f_i\left(1-\frac{x-x_i}{x_{i+1}-x_i}\right) + f_{i+1}\frac{x-x_i}{x_{i+1}-x_i}}_{\Delta y} \quad (5)$$

where:

$$x_i \leqslant x \leqslant x_{i+1}$$

Formula (5) represents a similarity transformation characterized by co efficients (a,b) and an added piecewise linear polynomial correction term for $\Delta x$ and for $\Delta y$.

Originally it was intended to use piecewise third-order polynomials, bu a suspicion about numerical instability resulted in the decision to incorp rate piecewise linear polynomials instead.

If the piecewise polynomial corrections consist of only one single piec then expression (5) reduces to an affine transformation where coefficient $e$ is responsible for differential scale, and coefficient $f$ for differential rotation.

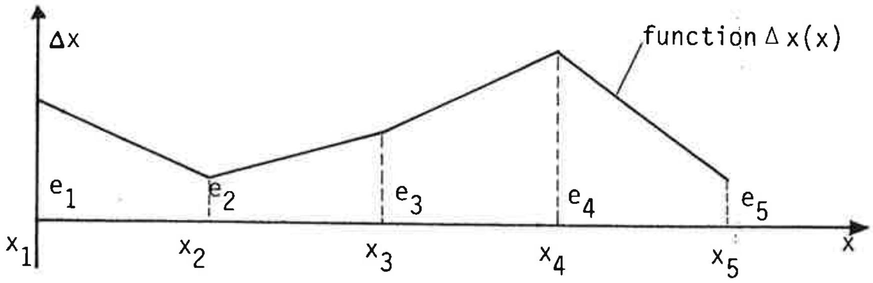


**Fig. 4.** Definitions for a piecewise polynomial to describe image errors $\Delta x$. For errors $\Delta y$, $e_i$ is replaced by $f_i$.

A simultaneous block adjustment with transformation (5) was programmed and called PIEPOL.

*Number of unknowns.* In the programmed algorithms, the normal equation matrix was always reduced to the actual transformation parameters, thus eliminating the unknown coordinates of tiepoints. A meaningful application of ANBLOCK or AFBLOCK to SLAR strips requires that these strips be broken up into pieces which overlap along the flightline. If $n$ indicates the number of SLAR strips in an adjustment, and $s$ the number of pieces into which a SLAR strip may be subdivided, then one obtained the following number $u$ of unknowns:

| ANBLOCK | AFBLOCK | PIEPOL |
|---|---|---|
| $u = n \cdot s \cdot 4$ | $u = n \cdot s \cdot 5$ | $u = (s+1) \cdot n \cdot 2 + 2 \cdot n$ |

and for $n = 20$, $s = 4$:

| | | |
|---|---|---|
| $u = 320$ | $u = 400$ | $u = 240$ |

These numbers indicate that a SLAR strip may be subdivided into more pieces with piecewise polynomials before reaching the same number of unknowns as in programs AFBLOCK or ANBLOCK.

## EVALUATION OF COMPUTATION METHODS

### *Description of experimental results*

A summary of the comparison of five methods for radargrammetric point determination is presented in Table I. The results are obtained from the simulated set of SLAR image data which were described in the section "Simulated SLAR imagery" and which do not show measuring errors nor a variation of flight height. Ten ground control points were assumed at equal distances along the perimeter. As mentioned before, meaningful application of the ANBLOCK or AFBLOCK programs requires that the original SLAR strips be broken into overlapping pieces. Table I gives the maximum number of pieces that could be computed on the UNIVAC 1108

TABLE I

RMS point discrepancies in mm (scale 1:400,000) between transformed and adjusted tiepoints and between radargrammetric and checkpoints; for explanation of methods, see the section on: "Planimetric SLAR block adjustment methods"

| | Sequential | | Simultaneous | | |
|---|---|---|---|---|---|
| | without splines | with splines | ANBLOCK (4 pieces) | AFBLOCK (4 pieces) | PIEPOL (6 pieces) |
| Tiepoints | 2.10 | 0.16 | 1.06 | 1.03 | 0.34* |
| Checkpoints | 1.90 | 1.02 | 1.51 | 1.59 | 1.23* |

* indicate that control point distribution is modified.

of JPL without going into specific storage compression for the normal equation matrices.

The major conclusion from Table I must be that with the given data and limitation in the size of normal equations sequential planimetric SLA block adjustment is superior to the simultaneous ones.

Two rms errors per adjustment algorithm are listed in Table I: rms residuals in tiepoints are the discrepancies between adjusted and transformed tiepoints; rms errors in checkpoints are the discrepancies between adjusted tie and true checkpoints. It is shown that in the studied arrange ment,sequential block formation with splines followed by linear least squares interpolation produces a block with very small discrepancies in tiepoints, smaller than any simultaneous method did produce; it also leaves the smallest discrepancies in checkpoints.

Table II presents the improvements in the performance of simultaneou methods obtainable when the strips are treated in a number of separate pieces. The rms discrepancies in tiepoints as well as in checkpoints are reduced significantly.

Analysis of distribution of ground control points was excluded as a separate objective of the study. But it was soon found, that simultaneou adjustment with piecewise polynomials (PIEPOL) was very sensitive to th parameter. The results degenerated rapidly in the case of unfavorable control distribution. When using more than two polynomial pieces, the distribution of control had to be modified: the points did not need to b distributed at equal distances along the perimeter, but one flightline (e.g. the first one) had to be specifically well controlled. The effect is indicat in Table II where both control distributions have been computed with th pieces per strip.

When using a sequential method, the formed block can be compared with checkpoints prior to and after external adjustment. The results are listed in Table III. Even without any use of control points, the formed

TABLE II

RMS point discrepancies in mm (scale 1:400,000) as in Table I; simultaneous adjustment methods; SLAR images broken in 1 to 4 pieces with overlap in flight direction

| Number of pieces | Tiepoints | | | Checkpoints | | |
|---|---|---|---|---|---|---|
| | ANBLOCK | AFBLOCK | PIEPOL | ANBLOCK | AFBLOCK | PIEPO |
| 1 | 2.10 | 2.09 | 2.09 | 1.90 | 2.63 | 2.74 |
| 2 | 1.70 | 1.66 | 1.71 | 1.69 | 1.97 | 2.01 |
| 3 | 1.38 | 1.37 | (1.17) 1.25* | 1.48 | 1.65 | (32.40) 2.15* |
| 4 | 1.06 | 1.03 | 0.77* | 1.51 | 1.59 | 1.33* |

* indicates that control distribution is modified; value in brackets is obtained with original distribution.

TABLE III

RMS point discrepancies in mm (scale 1:400,000), sequential adjustment methods; for explanation see text

| | Tiepoints | Checkpoints no interpolation | Checkpoints with interpolation |
|---|---|---|---|
| No splines | 2.10 | 1.90 | 1.10 |
| Splines weight $\frac{1}{1+d^2}$ | 0.16 | 1.30 | 1.02 |
| Splines equal weight | 0.42 | 1.29 | 0.98 |

block is comparatively undeformed. This, however, is largely a function of the initial strip of imagery: if this is undeformed, then the entire block will be. Deformations could only accumulate from systematic errors not accounted for in the mathematical model or from random errors (which were not present in the used set of data). But even in computations with data that did include random noise, there was no significant double summation effect. No large "pseudosystematic" deformations of the block were found that are so familiar in photogrammetric strips. But since overlap between SLAR strips is 60%, the block is less sensitive to a double-summation of random errors.

When using sequential splines, there is a choice of weight of each tiepoint in the computation of a joint: it is demonstrated that this weight is not critical (see rows 2 and 3 of Table III).

The above tables and conclusions are based on a block of SLAR images with image deformations only of the Schuler frequency type. Another block with measuring errors ($\sigma = \pm$ 70 to 150 m) was therefore used to study whether this would alter conclusions. The results appear in Table IV: the relative merits of SLAR block adjustment methods remain the same irrespective of the simulated block used in the computation.

*Discussion of results*

From the experiment, sequential block formation with splines followed by external adjustment appears to be the superior method. The statistical validity of this conclusion is of course limited, since it is based on results from only two simulated blocks. In addition, the particular method of formulating and programming the simultaneous adjustment methods is just one out of many alternatives. And finally, limitations of computer memor or programming resources did not allow exploitation of the full potential of simultaneous methods. But apart from these considerations, what explanation of the conclusions can be offered?

In the case of a SLAR block that only shows effects of Schuler period errors of inertial navigation, performance of the sequential splines method is solely a matter of "sampling": a high density of tiepoints produces a coherent block without cracks between strips; a sufficiently large number of ground control points along at least one flightline then allows to entirely eliminate block deformations. This indicates that under idealized co ditions; the above mentioned sequential method performs perfectly. These conditions show that the rms errors found in Table I for sequential spline are purely the result of insufficient density of tie and ground control points. A very good performance of the method must be expected since there were neither noise nor systematic errors in the imagery other than the Schuler type.

Actual images do show these errors, so that point determination cannot be perfect even with very high point densities: in Table IV, measuring errors and image deformations other than of the Schuler type have an eff Clearly, however, this effect is rather small and does not change the merit of the sequential spline method.

Method PIEPOL is a simultaneous least squares version of the sequentia spline method. Ultimately, PIEPOL should not be inferior, but rather superior in the case of random noise in the data; and it should be at least equivalent in the no noise case. But Tables I and IV do not agree with this expectation. Two reasons can be given for this: numerical limitations did not allow the increase in the number of polynomial pieces beyond six, while in the sequential method, the spline can be made as flexible as one wants; and a least squares solution of piecewise polynomia requires a sufficient density of control-points to prevent degeneration of results (see Table II, three pieces of polynomials). This second problem is nonexistent in a sequential version, to the effect, that it produced

TABLE IV

RMS discrepancies in mm at scale 1:400,000, using a SLAR block with simulated noise and variation in flight height in addition to Schuler-frequency

| | Sequential | | Simultaneous | | | | | |
|---|---|---|---|---|---|---|---|---|
| | no spline | with spline | ANBLOCK | | AFBLOCK | | PIEPOL | |
| | | | 1 piece | 4 pieces | 1 piece | 4 pieces | 1 piece | 4 pieces |
| Tie-points | 2.06 | 0.24 | 2.15 | 1.08 | 2.06 | 1.02 | 2.06 | 0.75* |
| Check-points | 2.00 | 1.10 | 1.90 | 1.56 | 2.63 | 1.64 | 3.07 | 1.26* |

* control distribution is modified.

superior results with both simulated blocks (Tables I and IV).

Sequential block formation without splines is not recommended, since its accuracy is poor (Tables I, III and IV). It is also not more economical than splines, since the additional effort for programming and computing splines is rather small.

The simultaneous equivalent to sequential block formation without splines is the ANBLOCK method, applied to SLAR strips that are not subdivided. Of course, this application of ANBLOCK is not appropriate and produces about the same result as its sequential counterpart. However, there is the question whether a subdivision of each SLAR strip into pieces could make the ANBLOCK a valid algorithm for the presented SLA[R] adjustment problem. The answer is no. The same applies for AFBLOCK which was hoped to be a refinement of ANBLOCK for the purpose. But AFBLOCK produces only slightly smaller cracks between strips due to the increase in the number of transformation parameters. Adjustment to ground control, however, is not better than with ANBLOCK.

This poor performance of the simultaneous methods, in particular ANBLOCK, as compared to the sequential method, was not expected. After all, a SLAR "block" resembles a photogrammetric strip, if the individual SLAR image takes on the role of the photogrammetric model; or if the SLAR image is broken into pieces, then a SLAR block could be compared to a photogrammetric block. And it is generally agreed upon in conventional photogrammetric strip and block adjustment, that simultaneous planimetric methods such as ANBLOCK are superior to sequential or even simultaneous methods employing polynomials. For an important reason, this conclusion cannot be extrapolated to the presented radargrammetric task: the required basic transformation of a SLAR strip is a polynomial one; similarity transformations of many pieces of a SLAR strip would only be an approximation.

In photogrammetry, however, basically a similarity transformation of models is required; and polynomial transformation of strips is the approximation. The roles of correct mathematical model and approximation are reversed.

Having established the requirement for, and superiority of polynomials for the purpose, what are the photogrammetric experiences of solving a simultaneous set of polynomials versus a sequential approach? To this question, a limited photogrammetric experience exists: Schut (1970) demonstrated that sequential formation of a photogrammetric block from individual strips, followed by external adjustment is equivalent to simultaneous polynomial block adjustment. This conclusion applies even more to the results of the presented study: Schut had only to deal with low order (2nd, 3rd degree) single polynomials, but meaningful SLAR block adjustment requires much higher order or piecewise polynomials. And the higher the order of the polynomial the more critical is the distribution of control or other external constraints to avoid degeneration of a simultaneous least squares solution.

## CONCLUSIONS

Two sequential methods of planimetric SLAR block adjustment, with and without splines, and three simultaneous methods according to the principles of least squares were evaluated. A limited experiment with simulated SLAR images indicated that sequential planimetric SLAR block formation followed by linear prediction using ground control points produces results which could not be obtained with simultaneous methods such as ANBLOCK.

This conclusion is less unexpected when the fact is recognized that a SLAR image strip has a simpler error behavior as compared to a photogrammetric strip; and that on the other hand, the basic transformation of a SLAR strip is a polynomial one, whereas a similarity transformation of pieces is an approximation. This is contrary to the photogrammetric case. However, these conclusions are only valid under the constraints encountered in the study concerning computer memory and programming resources.

Use of splines significantly improves the results of internal adjustment by sequential block formation. Internal adjustment (block formation) without splines should not be considered. Simultaneous methods with ANBLOCK (similarity transformations), with transformations with differential scale, and especially with transformations with splines, approach the performance of the sequential method with increasing subdivision of each SLAR image strip into separate units. The computational efforts, however, are much larger in a simultaneous least squares adjustment than in sequential block formation followed by external adjustment. In addition, control distribution and numerical stability of a least squares solution are critical in a simultaneous method.

Consequently, the study suggests that sequential block formation with splines followed by external interpolative adjustment should be used as an inexpensive means of satisfactory planimetric point determination from SLAR images of flat terrain. It is conjectured that this conclusion might very well also apply to other sets of continuous imageries, such as orbital or airborne scanner images (ERTS).

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