THEORETICAL FIELD ANALYSIS FOR SUPERFERRIC ACCELERATOR MAGNETS USING PLANE ELLIPTIC OR TOROIDAL MULTIPOLES AND ITS ADVANTAGES

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Abstract

FAIR will build a set of accelerators and storage rings at GSI Darmstadt. Nearly all of them transport beams of elliptical shape (SIS 100, CR, NESR, RESR, SuperFRS). Magnetic field calculations as well as magnetic measurements provide precise field information, which is used to improve the properties of machine using numerical simulations. We had developed elliptical multipoles fulfilling the Laplace equation which enable us to describe the field within the whole aperture consistently. Now the representation of these has been simplified considerably as compared to earlier ones. Meanwhile we found analytical expressions to derive circular multipoles directly from the elliptic multipoles. We illustrate the advantage of this data representation on SIS 100 magnet data.

INTRODUCTION

Conventional magnets found in accelerators provide typically a rectangular aperture and for accelerators of small circumference (up to a radius of a few tens of meter) these are also of curved shape. These magnets were typically measured using search coil probes.

The SIS 100 synchrotron, the core component of the FAIR facility, uses superferric magnets and a cable with superconductors wound around a tube. As these synchrotron magnets are housed in a interconnected cryostat, introducing additional magnetic elements requires to warm up the machine, cut the connections, and reweld them afterwards as well as a cooling them down again. Therefore the field properties have to be fully understood right from the beginning and a sufficently accurate and concise field description is required.

A common type of such a description is an expansion in plane circular multipoles [1]. We generalized the concept by introducing plane elliptic multipoles [2, 3] as particular solutions of the potential equation in elliptical coordinates. The complete set of basis function needed for an expansion of an arbitrary solution is only a true subset of all these solutions. The advantage is that the reference curve is an ellipse accomodated to the rectangular transverse gap cross section, which covers a larger area than that of a possible reference circle. This expansion prooved to be superior to the circular expansion. A method to extract the elliptic expansion coefficients from experimental data acquired by rotating coils has been proposed and tested numerically[3].

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As the SIS 100 dipoles are curved we started to investigate this question by another generalization defining plane toroidal multipoles. These are again particular solutions of the potential equation in local toroidal coordinates. With the separation method analytical solutions in these coordinates can be calculated approximately using only power series expansions in the inverse aspect ratio (= the fraction minor/larger radius), cf. eq.(10). This method is well-known in the microwave theory for curved waveguides [4]. The multipole solutions so obtained show very clearly the effects of the curvature and their magnitude, which is of the order of the inverse aspect ratio.

THEORY

The SIS 100 magnets provide an elliptic aperture and the dipoles are curved to follow the beam sagitta. For a thorough understanding of the measurement a solution of $\Delta\Phi = 0$ is required.

Circular Multipoles

The two field components B_x, B_y of a plane irrotational source-free static magnetic field are combined to a complex field $\mathbf{B} := B_y + iB_x$ depending on the complex variable $\mathbf{z} := r\, e^{i\theta}$; they are expanded in circular multipoles

$$\mathbf{B} = \sum_{m=0}^{M} \mathbf{C}_m\, (r/R_R)^m\, e^{im\theta} \tag{1}$$

$$\mathbf{C}_m = \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathbf{B}(\mathbf{z} = R_R\, e^{-im\theta})\, e^{-im\theta} d\theta, \tag{2}$$

with M the number of multipoles used. The expansion coefficients may be computed from field values given along the reference circle $r = R_R$ as indicated in eq.(2).

Elliptic Multipoles

A reference ellipse with semi-axes $a > b$ accomodated to a rechtangular gap covers a larger domain than a reference circle of radius R_R. The excentricity e specifies the corresponding elliptic coordinates η, ψ, [5, 3, 2]

$$x = e\cosh\eta\cos\psi, \quad 0 \le \eta \le \eta_0 < \infty; \tag{3}$$

$$y = e\sinh\eta\sin\psi, \quad -\pi \le \psi \le \pi. \tag{4}$$

$\eta_0 = \tanh^{-1}(b/a)$ gives the reference ellipse. A plane irrotational source-free field is expanded w.r.t. the complete system of elliptic multipoles as

$$\begin{aligned}\mathbf{B} &= \sum_{n=0}^{M} \mathbf{E}_n \ \cosh[n(\eta + i\psi)]/\cosh(n\eta_0), \\ \mathbf{E}_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} \mathbf{B}(\mathbf{z} = e\cosh(\eta_0 + i\psi)) \ \cos(n\psi) \ d\psi.\end{aligned} \tag{5}$$

The expansion coefficients $\mathbf{E}_n$ may be computed from field values given along the reference ellipse (5).

The two sets of expansion coefficients belonging to the same $\mathbf{B}$ may be converted to each other using

$$\mathbf{E}_n/\cosh(n\eta_0) = \sum_{m=0}^{M} \mathbf{C}_m \ \beta^m \ d_{mn}, \tag{6}$$

$$2\ \mathbf{C}_m \ \beta^m = \sum_{n=0}^{M} \mathbf{E}_n/\cosh(n\eta_0) \ c_{nm} \tag{7}$$

with $\beta := e/(2R_R)$. The transformation matrices $D = (d_{mn})$ and $C = (c_{nm}) = D^{-1}$ are given by

$$d_{mn} = [1 + (-1)^{m+k}] \begin{pmatrix} m \\ (m-k)/2 \end{pmatrix}. \tag{8}$$

The elements of C may be found by symbolic or numeric inversion of D; closed expressions have been given elsewhere [2]; in [6] they are computed by recurrences.

Toroidal Multipoles

In a curved magnet a torus segment ($|\varphi| \leq \varphi_0$) is introduced as a reference volume. Dimensionless local toroidal coordinates are defined by

$$X + iY = R_C \ h \ e^{i\varphi}, \ \ Z = R_R \sin\vartheta, \ \ h = 1 + \epsilon\rho\cos\vartheta. \tag{9}$$

R_R (R_C) are the minor (major) radii of the torus.

$$\epsilon := R_R/R_C \tag{10}$$

is the inverse aspect ratio, on which the curvature effects depends. As $\epsilon << 1$ working with power series in ϵ is a useful approximation scheme. The centre of the fundamental Cartesian system (X, Y, Z) coincides with that of the torus, Z is normal to the equatorial plane. The quasi-radius $R_R \cdot \rho$, $0 \leq \rho \leq 1$, is the normal distance of the field point from the centre circle; the poloidal angle $-\pi \leq \vartheta \leq \pi$, is around the centre circle; the toroidal angle $-\pi \leq \varphi \leq \pi$ agrees with the common azimuth, cf. [7], [4]. Only toroidally uniform fields are considered; their field components B_ρ, B_θ are confined to the planes φ = const. and are independent of φ.

The potential equation independent of φ is solved by an approximate R-separation. Thus the approximate multipole solution for the potential is (m = integer)

$$\begin{aligned}\Phi_m &= \rho^{|m|} \ e^{im\vartheta} \\ &\quad -\frac{\epsilon}{4} \ \rho^{|m|+1}\big(e^{i(m+1)\vartheta} + e^{i(m-1)\vartheta}\big) + O(\epsilon^2).\end{aligned} \tag{11}$$

So the curvature adds just the two adjacent multipoles; the magnitude of these admixture is not larger than $\epsilon/2$. Expressions for the corresponding magnetic fields have been derived as well as their orthogonality relations. This permits us to give the field expansion w.r.t. the basis fields and to calculate the expansion coefficients for a field given along the reference circle. A report will be published. We prepared a theory which will permit us to determine these coefficients from the Fourier components of the Voltage induced in a coil rotating in such a curved magnet.

APPLICATION

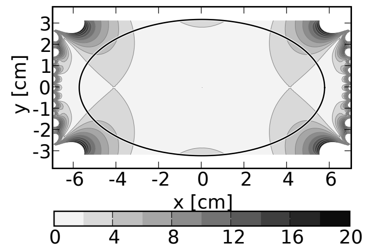
The formulae described above were used to analyse all the magnet data calculated for the SIS 100 main magnet designs. The field quality was calculated for the **C**urved **S**ingle **L**ayer **D**ipole with **8** turns [3], the dipole design chosen for the main dipole for the SIS 100 machine of FAIR. The field quality of this magnet is calculated by

$$\mathbf{\Delta B}(\mathbf{z}) = (\mathbf{B}(\mathbf{z}) - \mathbf{B}(\mathbf{0})/\,\mathbf{B}(\mathbf{0}) \cdot \ 10^4\ . \tag{12}$$

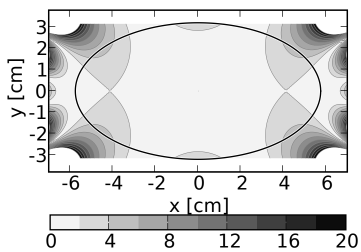
They are now applied to fields to demonstrate that all these steps are necessary to interpolate the field within the ellipse with a precision of better than the maximum tolerable field deviation of 600 ppm or 6 units (1 unit corresponds to 100 ppm). The original distribution is given in Fig. 1(a) at a current of $873kA$ yielding a field of $\approx \ 0.13T$. The field was taken along the ellipse and the elliptic multipoles were calculated as defined in (5). Using the first 20 coefficients the field was interpolated within the aperture (see Fig. 1(b)). The naked eye can not see any difference to the original data (Fig. 1(a)). The original field was subtracted from the interpolated one. One can see from Fig. 1(e) that this difference is well below half a unit and thus sufficiently precise. Normally circular multipoles are used. So we calculated them using a Fourier Transform of the data along a circle. Again the interpolation data was calculated (see Fig. 1(d)) and the difference to the original data (see Fig. 1(g)) using the first 15 coefficients. One can see that the interpolation works well within the circle but outside the circle soon the errors get unacceptably large. The difference outside of the circle is even larger if more coefficients are used. At last the circular multipoles were calculated from the elliptic ones as described in (8) (see Fig. 1(c) for the interpolation and Fig. 1(f)) for the difference). One can see that contrary to the circular interpolation, this interpolation works even outside the circle and within the whole ellipse.

CONCLUSION

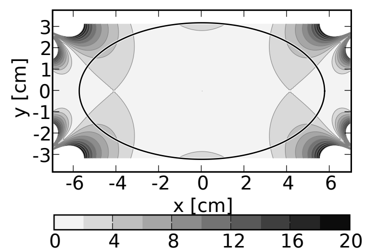
Elliptic multipole expansions for a plane irrotational, source-free, static magnetic field were demonstrated in a domain bounded by an ellipse as reference curve similar as for circular multipoles within a circle. In both cases the expansion coefficients of the complex field can be computed from the field given along the reference curve. The ellipse covers a larger area in the gap and thus the convergence



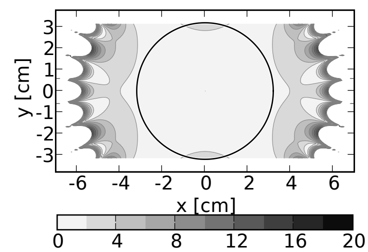
(a) Deviation of original data from a pure dipole field



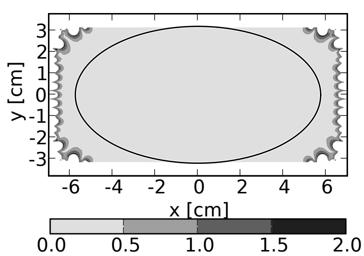
(b) Deviation of field computed by elliptic expansion from dipole field



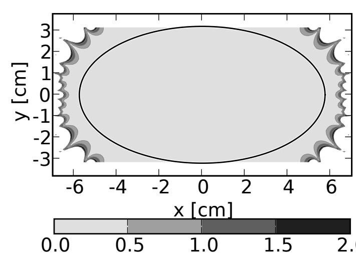
(c) Deviation of field computed by circular expansion with circular coefficients converted from elliptic coefficients, eq.(8)



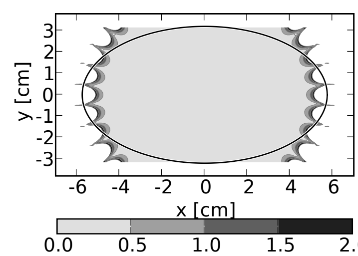
(d) Deviation of field computed by circular expansion from dipole field



(e) Difference between original field data and those computed by elliptic expansion



(f) Deviation of original field data from data computed as described at left



(g) Difference between original field data and those computed by circular expansion

Figure 1: Test of the interpolation for the CSLD8b at a current of $873\,kA$ and a field of $\approx 0.13\,T$. The field B_y in the aperture is plotted. The gray indicates the absolute value of the deviation (in units). The original data are given on top. The upper row shows the data as reconstructed using the interpolation and the lower columns shows the absolute value of the difference between the reconstructed and the original data.

properties are better for the elliptic expansion. We demonstrated the validity of the elliptic and circular coefficients for the 2D-field of the Curved Single Layer Dipole with 8 turns (CSLD-8b). We showed that only these sets allow to reconstruct the field with a precision of better than 1 unit within the ellipse. Plane toroidal multipoles have been derived to expand the field in curved magnets. These show that the curvature mixes the two neighbouring mulipoles to the main circular multipole. The magnitude of these is of the order of the inverse aspect ratio, which is the transverse dimension of the gap divided by the curvature radius.

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