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Influence of Incompressibility on Different Wave Types in Porous Media

There are three wave types in poroelastic continua, the fast compressional wave, with solid and fluid moving in-phase, the shear wave, and the second (slow) compressional wave, which has no equivalent in elastic materials, with solid and fluid moving in opposite directions. The fast compressional wave propagates with infinite speed if both constituents are modelled incompressible.

Numerical results of BEM calculations showing the influence of incompressible constituents will be presented as well as elements employing different shape functions for the solid displacements and the pore pressure.

1. BEM Formulation

The Biot's theory is used to model the constitutive behaviour of a poroelastic continuum. For some materials, e.g., soil, the compressibility of the constituents itself is much smaller than the compressibility of the structure. In these cases, it is sufficient to approximate both the fluid and solid constituents as incompressible. The assumption of incompressible constituents in a two-phase material is given if $\frac{K}{K_s} \ll 1$ and $\frac{K}{K_f} \ll 1$ holds with K_s and K_f denoting the compression moduli of the solid grains and the fluid, respectively, and K the bulk compression modulus.

The solid displacements u_i and pore pressure p are chosen as a sufficient set of independent variables. The derivation of the integral equation follows the procedure for the compressible case and, therefore, is not explained here. Further, also the time stepping procedure is implemented, as in case of compressible constituents, using the Convolution Quadrature Method, establishing a time domain formulation using Laplace domain fundamental solutions. Details on this BE formulation may be found in [1]. During the spatial discretization, the quantities are approximated using the nodal values $u_i^{ef}(t)$, $p^{ef}(t)$ and the shape functions ${}^{u}N_e^f(\mathbf{x})$, ${}^{p}N_e^f(\mathbf{x})$ corresponding to the node f of element e. For displacements u_i and pore pressure p, this gives

$$u_i(\mathbf{x},\mathbf{t}) = \sum_{e=1}^{E}\sum_{f=1}^{F} {}^{u}N_e^f(\mathbf{x})\, u_i^{ef}(t)\,, \quad p(\mathbf{x},\mathbf{t}) = \sum_{e=1}^{E}\sum_{f=1}^{F} {}^{p}N_e^f(\mathbf{x})\, p^{ef}(t)\ . \tag{1}$$

The tractions t_i and flux q are handled in the same way. The simplest choice are isoparametric elements, i.e., the ansatz functions identical for all quantities and the geometry. Another option, common in finite elements, is to choose the ansatz for p, q one degree lower than for u_i and t_i, e.g., ${}^{u}N_e^f(\mathbf{x})$ linear and ${}^{p}N_e^f(\mathbf{x})$ constant. This has been added to the BEM implementation (details may be found in [2]).

2. Example: 2D Soil Column

A poroelastic column, see Fig. 1, is discretized with 32 isoparametric linear (`li-2D`) and linear-constant (`lk-2D`) elements. The time step size used is $\Delta t = 0.001\,\mathrm{s}$. The bottom is fixed and the top surface is loaded by a vertical total stress vector $t_y = -1\,\mathrm{N/m^2}\,\mathrm{H}(\mathrm{t})$. The remaining surfaces, i.e., the sidewalls, are traction free. The pore pressure is assumed to be zero on the top, i.e., the surface is permeable, and the sides and bottom are impermeable. The material data of soil can be found in table 1. The displacement in the middle of the top surface is plotted versus time in Fig. 2. The element type `lk-2D` shows less numerical damping than the `li-2D`. So, it can be concluded that the elements combining linear and constant ansatz functions improve the results.

3. Example: 3D Halfspace

In order to demonstrate the effect of modelling the constituents incompressible, a poroelastic half space is considered. The data for the two different materials used, a water saturated rock (Berea sandstone) and a water saturated coarse

Table 1: Material Data

	$K, G\ [\frac{N}{m^2}]$		$K_s, K_f\ [\frac{N}{m^2}]$		$\rho, \rho_f\ [\frac{kg}{m^3}]$		ϕ	$R\ [\frac{N}{m^2}]$	α	$\kappa\ [\frac{m^4}{Ns}]$
Berea sandstone	$8{\cdot}10^9$	$6{\cdot}10^9$	$3.6{\cdot}10^{10}$	$3.3{\cdot}10^9$	2458	1000	0.19	$4.7\cdot10^8$	0.778	$1.9{\cdot}10^{-10}$
Soil (coarse sand)	$2.1{\cdot}10^8$	$9.8{\cdot}10^7$	$1.1{\cdot}10^{10}$	$3.3{\cdot}10^9$	1884	1000	0.48	$1.2\cdot10^9$	0.981	$3.55{\cdot}10^{-9}$

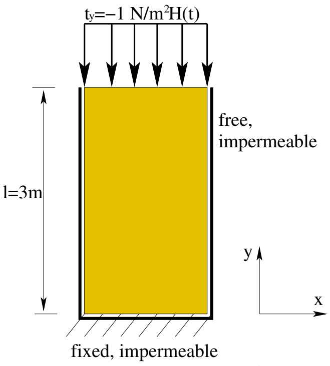

Figure 1: 2D column

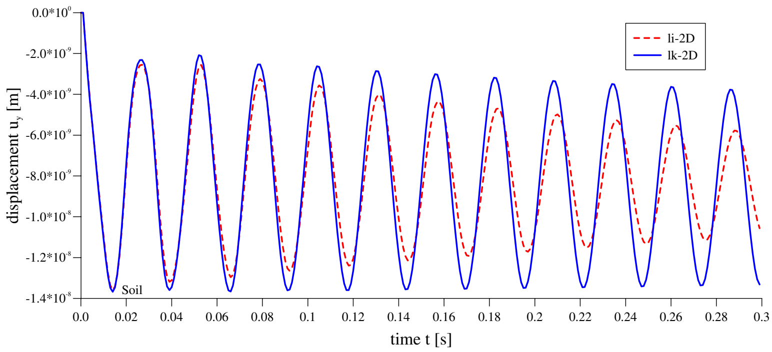

Figure 2: Displacements at top versus time: different shape functions

sand (soil), can be found in Tab. 1. A long strip (6 m × 33 m) is discretized with 396 triangular linear elements on 242 nodes. The time step size used is $\Delta t = 0.00016$ s in case of rock and $\Delta t = 0.00064$ s in case of soil. The modelled half space is loaded by a vertical total stress vector $t_z = -1000\,\mathrm{N/m^2}\,\mathrm{H}(t)$ on a triangular area of $1\,\mathrm{m}^2$ and the remaining surface is traction free. The pore pressure is assumed to be zero all over the surface, i.e., the surface is permeable.

Before examining the results, it may be convenient to look at the ratios of the compression moduli. There is $K/K_s = 0.22, K/K_f = 2.42$ for the rock and $K/K_s = 0.019, K/K_f = 0.064$ for the soil. Hence, the incompressible modelling can be expected to fail for the rock and to give good results for the soil, which is confirmed by the results given in Fig. 3. There, the pore pressure p at a point 3m under the loaded area is plotted versus time t for both

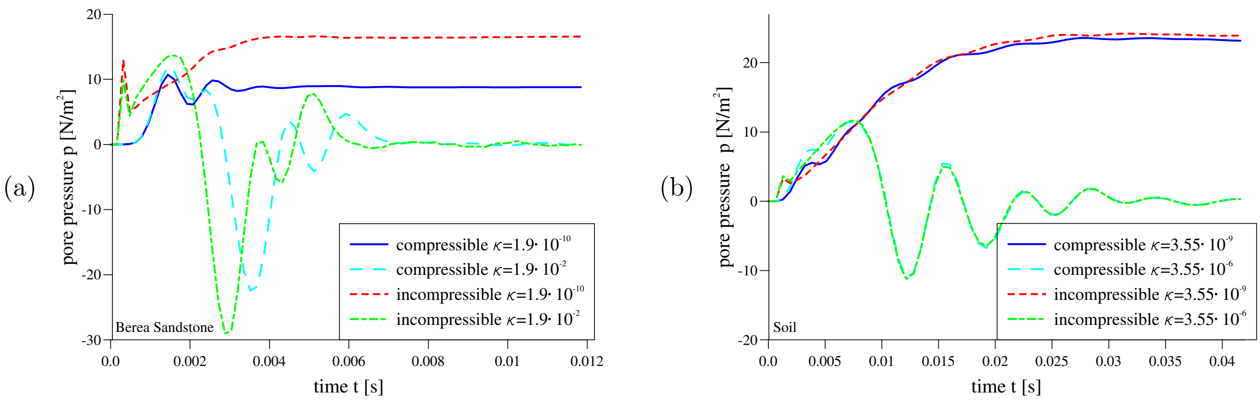

Figure 3: Pore pressure in 3m depth versus time for berea sandstone (a) and soil (b)

the incompressible and compressible model. Clearly, there are large differences for the rock whereas for soil both results are almost indistinguishable. Also, from the incompressible rock results it can be observed that the arrival of the fast compressional wave, the first deviation from zero, tends to zero. The results with increased permeability κ show that the slow compressional wave speed is changed, which also corresponds to the theory.

Finally, it should be remarked that the usage of the incompressible modelling brings a speedup of about $\sim 20\%$ compared to the compressible model. Therefore, for materials fulfilling the incompressibility assumptions (e.g., loose grain with fluid), the incompressible model can be recommended.

Acknowledgements

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4. References

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