Comment on "Scaling of asymmetric magnetic reconnection: General theory and collisional simulations" [Phys. Plasmas 14, 102114 (2007)]

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Recently, Cassak and Shay [Phys. Plasmas **14**, 102114 (2007)] proposed a new scaling for asymmetric reconnection referred to as Sweet–Parker-type scaling. We set into question the derivation of those scaling laws. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3083261]

Starting from the basic equations for mass, momentum, and energy conservation, as well as the equation for the electric field [Eqs. (6)–(9) in their paper], the following expression for the outflow velocity is derived:

$$v_{\text{out}}^2 \sim \frac{B_1 B_2}{4\pi}\frac{B_1+B_2}{\rho_1 B_2+\rho_2 B_1}, \tag{1}$$

which reduces to the geometrical mean for equal densities

$$v_{\text{out}} \sim \sqrt{\frac{B_1 B_2}{4\pi\rho}} \quad \text{for} \quad \rho_1=\rho_2=\rho. \tag{2}$$

This result for the outflow velocity is, in our opinion, false and therefore also the subsequent formulas based on this result are at least questionable, in particular, the expression for the outflow density ρ_{out} [Eq. (17)]. This result has been used in a series of subsequent publications[1–3] and the same result has also been obtained in a similar way in a paper by different authors at about the same time.[4]

If we repeat their analysis based on Eqs. (6), (8), and (9) of their paper assuming for simplicity $\rho_1=\rho_2$, we arrive at the following set of equations:

$$(v_1+v_2)L = v_{\text{out}}\delta, \tag{3}$$

$$B_1 v_1 = B_2 v_2, \tag{4}$$

$$\left[\left(\frac{B_1^2}{8\pi}+p_1\right)v_1+\frac{B_1^2}{8\pi}v_1+\left(\frac{B_2^2}{8\pi}+p_2\right)v_2+\frac{B_2^2}{8\pi}v_2\right]L = \left[\frac{\rho v_{\text{out}}^2}{2}+\left(\frac{B_{\text{out}}^2}{8\pi}+p_{\text{out}}\right)+\frac{B_{\text{out}}^2}{8\pi}\right]v_{\text{out}}\delta-\frac{v_{\text{out}}B_{\text{out}}}{4\pi}B_{\text{out}}\delta. \tag{5}$$

If we further assume total pressure balance [component normal to the layer of their Eq. (7)],

$$\frac{B_1^2}{8\pi}+p_1=\frac{B_2^2}{8\pi}+p_2=\frac{B_{\text{out}}^2}{8\pi}+p_{\text{out}}, \tag{6}$$

and therefore,

$$\left(\frac{B_1^2}{8\pi}v_1+\frac{B_2^2}{8\pi}v_2\right)L=\left(\frac{\rho v_{\text{out}}^2}{2}-\frac{B_{\text{out}}^2}{8\pi}\right)v_{\text{out}}\delta. \tag{7}$$

For $B_{\text{out}}=0$ this equation reduces to the formula in the commented paper. However this is only true in special "symmetric" cases, i.e., antiparallel magnetic fields and equal Alfvén velocities in the inflow regions.[5] If there is some asymmetry either in magnetic field and/or densities, B_{out} in the outflow region in general turns out to be of the same order as the magnetic field in the inflow region.

Because of the additional unknown we need further equations. Equation (9) of their paper relates the electric field across the layer. Expressed through the normal and tangential components, this reads

$$\{v_n B_t - v_t B_n\}=0, \tag{8}$$

where the curly bracket means $\{F\}\equiv F_{\text{in}}-F_{\text{out}}$. Also from their Eq. (7) we get

$$\left\{\rho v_n v_t-\frac{B_n B_t}{4\pi}\right\}=0. \tag{9}$$

These two relations my be combined to give

$$v_n = \pm\frac{B_n}{\sqrt{4\pi\rho}}, \tag{10}$$

where the sign is the sign of mass flux times magnetic flux through the boundary (flow and field in the same, "+," or the opposite directions, "−"). If this relation is used, we obtain from Eqs. (8) and (9)

$$\left\{\pm\frac{B_t}{\sqrt{4\pi\rho}}-v_t\right\}=0. \tag{11}$$

Therefore, if we specify $\mathbf{B}_1=(-B_1,0)$, $\mathbf{B}_2=(B_2,0)$, $B_1>0$, $B_2>0$, we have to take on the right hand side of the reconnection line the + sign for upper boundary, and the − sign for lower boundary of the outflow region. The magnetic field within the outflow region then is

$$B_{\text{out}}=-B_1+\sqrt{4\pi\rho}v_{\text{out}}=B_2-\sqrt{4\pi\rho}v_{\text{out}}. \tag{12}$$

Now we can finally solve for outflow field and flow

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$$B_{\text{out}} = \frac{B_2 - B_1}{2}, \tag{13}$$

$$v_{\text{out}} = \frac{B_1 + B_2}{2\sqrt{4\pi\rho}}. \tag{14}$$

This is, of course, consistent with energy relation (7). The outflow velocity therefore is not the geometrical mean but the arithmetic mean of the inflow Alfvén velocities. These two values can significantly differ for high asymmetries.

In this comment we repeat the incompressible analysis for asymmetric reconnection. The complete set of equations have also been analyzed in detail for the compressible asymmetric case of reconnection.[5] This sort of one dimensional mass, energy, and stress balance analysis is also part of the more general time dependent case analyzed in Ref. 6 even in higher dimensions with finite reconnection line,[7] and additional shear in velocity.[8]

In this time dependent analysis there appears as a typical velocity also the quadratic mean

$$\sqrt{\frac{B_1^2 + B_2^2}{8\pi\rho}}. \tag{15}$$

This velocity is related to the propagation speed of disturbances along the perturbed surface (named magnetoacoustic surface wave in Ref. 9). We do not agree with the statement that "little was known about asymmetric reconnection until recently"[3] because several parts, in particular, stress balance, have been studied in detail in the past.[5–8,10–12]

Physically, as can be seen from Eq. (10), there is an Alfvén wave standing in the inflow and as a consequence, Alfvén and plasma velocities are related by stress balance as given by Eq. (12) and those waves are of switch-off type only for very specific symmetries in the inflow regions.

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