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Hysteretic Friction of a Sliding Rubber Element

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Friction, especially friction of elastomers, can cause acoustic problems like noise, squeal and comfort drawbacks like vibrations and wear. Therefore, rubber friction affects the function of many products in technical applications, e.g. seals, belts and tires. It can be classified according to different physical phenomena like adhesion, hysteresis, cohesion and viscous friction, see [3]. The topic of this paper is hysteresis friction of rubber that is caused by the energy dissipation due to internal material damping during the process of deformation. The deformation itself occurs during the sliding of a rubber element across the micro-scaled asperities of a rough surface. In this paper, the sliding process of a rubber element over real surfaces is simulated in time domain and compared to experiments.

1 Theory of Hysteretic Friction

The simulation of the friction process is divided into two parts. First, it is necessary to build a material model of rubber that comprises the required properties including relaxation and creeping. This is realized by an extended Maxwell model consisting of linear spring and damper elements, see Figure 1. The number of Maxwell branches determines the quality of the model but also the number of parameters. The system parameters are found by a special fit algorithm applied to the experimentally obtained frequency dependent complex modulus.

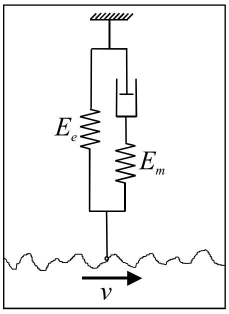

Fig. 1 Spring-damper-system

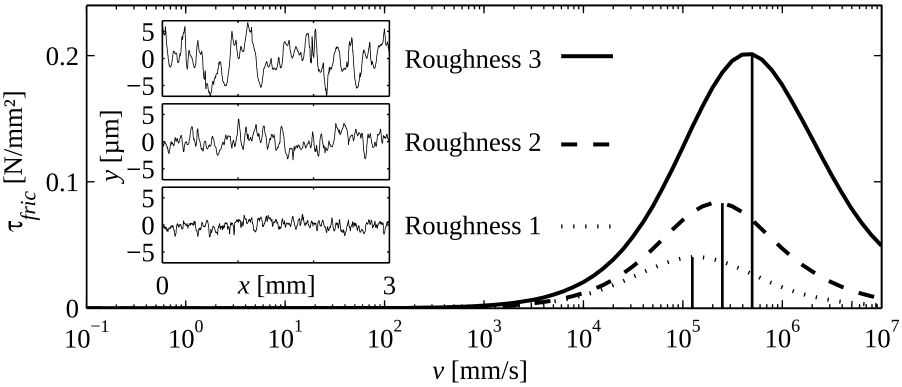

Fig. 2 Calculated friction shear stress dependent on measured surface roughnesses

Secondly, the simulation requires the excitation by the surface roughness to calculate the response as well as the corresponding normal stress and the energy dissipation. The specific mean energy loss w_{diss} during the stationary sliding process is equated to the applied specific mean energy w_{fric} caused by the friction forces. The mean friction shear stress τ_{fric} is calculated as an average of the specific dissipated energy multiplied by the mean layer thickness z_p of the excited rubber volume and divided by the sliding distance x_{fric},

$$\tau_{fric} = \frac{z_p}{x_{fric}}\, w_{fric} \qquad \text{with} \qquad w_{fric} = w_{diss} = \int_0^T \sigma\dot{\varepsilon}\,\mathrm{d}t\,. \tag{1}$$

Here, the mean layer thickness z_p of the rubber surface is an unknown parameter. The mean friction shear stress τ_{fric} is calculated for different stationary relative velocities v to find the relation $\tau_{fric}(v)$. The coefficient of friction is found as the ratio of $\tau_{fric}(v)$ and the mean normal stress $\sigma(v)$,

$$\mu_{fric}(v) = \frac{\tau_{fric}(v)}{\sigma(v)}\,. \tag{2}$$

The model is validated by a harmonic excitation due to a sinusoidal surface. Finally, the simulation that includes the material model and surface roughnesses based on real needle measurements is performed. Figure 2 shows the friction shear stress simulations for different roughness measurements depending on the relative velocity v of the contact.

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2 Theory of Klüppel and Heinrich

The theory of Klüppel and Heinrich is based on the energy loss during the sliding process using a similar approach as in this paper, see [2]. However, the description of the physical process is given in frequency domain using a Fourier transformation including the power spectral density $S(\omega)$ of the roughness. In this case the hysteretic friction coefficient reads

$$\mu_{KH}(v) = b\,\frac{z_p}{8\pi^2\sigma_0 v}\int \omega\, E''(\omega)\, S(\omega)\,\mathrm{d}\omega\,. \tag{3}$$

Here, the power spectral density $S(\omega)$ is weighted by the loss modulus $E''(\omega)$ of the rubber material.

3 Comparison of Simulation Results and Experiments

An important result of the simulations is the dependence of the hysteretic friction coefficient $\mu_{fric}(v)$ on the relative velocity v neglecting the influence of temperature. In the simulations as well as in the experiments, the same rubber specimen and the same surface roughness are used. The experiment has been made on a tribometer test rig with a pin on disc configuration regarding experiences shown in [1]. Here, an unfilled elastomer of SBR and a corundum surface has been used. The average measurement values of the stationary friction coefficient are shown in Figure 3 together with simulation results. In the experiments the contact velocity has been changed in steps. For this velocity range the temperature influence can be neglected. The results in Figure 3 show a good agreement between experiments and simulations.

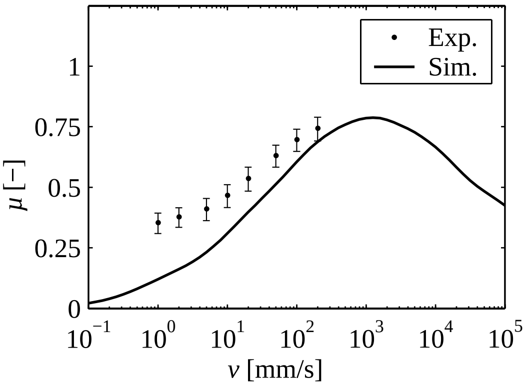

Fig. 3 Comparison of simulation results and experiments

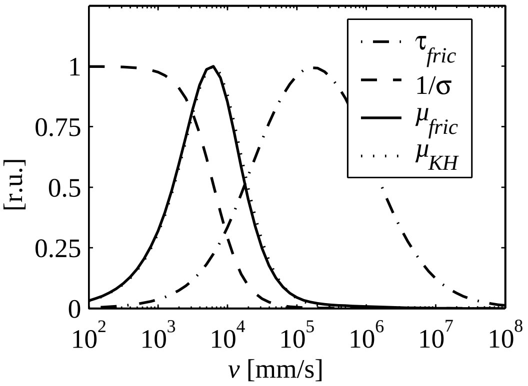

Fig. 4 Comparison with the theory of Klüppel and Heinrich

4 Conclusions

The advantage of the described simulation method is the approximation of the real physical hysteretic friction process and its description in time domain. It allows to use surface measurements directly as an input for the simulation and to implement non-linear rubber behavior into the model if necessary. The dependence of the friction coefficient on the parameters velocity and roughness of a hard surface has been investigated. The results have been compared to an alternative theory of Klüppel and Heinrich which is formulated in frequency domain and shows a good congruence with respect to the velocity dependence of the hysteretic friction coefficient, see Figure 4. Furthermore, the simulation has been compared to real friction experiments for the same material and roughness. The results show a good agreement and give the possibility to interpret the influence of additional adhesion effects which have not been discussed in this paper.

Acknowledgements

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