MHD stability analysis for Tokamaks

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Motivation

In recent studies of the interaction of low frequency resonant magnetic field perturbations it was demonstrated that MHD theory has strong limitations in its applicability for modern Tokamak parameter range (Heyn et al. 2008) . Namely, the radial scale of resonant layers in plasma is comparable to the ion Larmor radius. Therefore it is interesting to check the MHD results for various instabilities connected with resonant magnetic flux surfaces (kink modes, resistive wall modes) using the kinetic approach.

The purpose of this work is to adapt the kinetic code of (Heyn et al. 2008) to the problem of finding the global modes and increments for these modes. First results of such calculations are compared to ideal MHD calculations described in (Heyn et al. 2008).

Approach to MHD instability analysis

To model MHD-plasma instabilities, ideal MHD-equilibrium equations in one dimensional screw pinch geometry are used,

$$\nabla p = \frac{1}{c}\,\mathbf{j_0} \times \mathbf{B_0} \quad , \tag{1}$$

$$\nabla \times \mathbf{B_0} = \frac{4\pi}{c}\mathbf{j_0} \quad , \tag{2}$$

$$\frac{\partial \rho}{\partial t} + \nabla\left(\rho v\right) = 0 \quad . \tag{3}$$

In screw pinch equilibrium

$$\frac{\partial}{\partial r}\left(p + \frac{B_0^2}{8\pi}\right) + \frac{B_\theta}{4\pi r} = 0\,, \tag{4}$$

the equilibrium field $\mathbf{B_0}$ is assumed to have no radial component $B_r = 0$, $\mathbf{B_0} = (0, B_\theta, B_z)$.

The plasma perturbation is treated as an initial value problem which is expressed by the use of a plasma displacement vector $\xi\,(r,t)$ where

$$\xi\,(r,0) \quad = 0\,,$$

$$\frac{\partial \xi\,(r,0)}{\partial t} = v_1\,(r,0) \quad . \tag{5}$$

Linearisation of the equations for small perturbations, formulation as an eigenvalue problem and the application of the variational principle in order to minimise the potential energy of the investigated system, yield the normal mode eigenequation (6) for the eigenfunctions (Freidberg 1970),

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[A\,(r,\gamma)\,\frac{\mathrm{d}}{\mathrm{d}r}\,(r\xi_r)\right] - C\,(r,\gamma)\,\xi_r = 0\,, \tag{6}$$

$$A\,(r,\gamma) = \frac{r\left(\gamma^2\rho + \frac{F^2}{4\pi}\right)}{k^2r^2 + m^2}\,, \tag{7}$$

$$B\,(r,\gamma) = \gamma^2\rho + \frac{F^2}{4\pi} - \frac{2B_\theta}{4\pi}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{B_\theta}{r}\right) + \frac{2mr}{4\pi}\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{FB_\theta}{r\,(k^2r^2+m^2)}\right] - \frac{4k^2F^2B_\theta^2}{16\pi^2\,(k^2r^2+m^2)\left(\gamma^2\rho + \frac{F^2}{4\pi}\right)}\,, \tag{8}$$

$$F = kB_z + \frac{mB_\theta}{r} \tag{9}$$

Here, m is the poloidal mode number, γ is the real part of the eigenmode frequency, k is the z-component of the wave vector, ρ is the mass density and function F describes resonances of the instability modes.

The Boundary conditions for the radial component of ξ are $\xi_r\,(0) = 1$, $\xi_r\,(r_w) = 0$, at the magnetic axis and at the wall position r_w, respectively

Kinetic instability analysis

For kinetic instability modelling the Vlasov equation

$$\frac{\partial f}{\partial t} + \nabla\cdot(f\mathbf{v}) + \frac{F}{m}\frac{\partial f}{\partial v} = \left(\frac{\partial f}{\partial t}\right)_C \tag{10}$$

with a Coulomb collision term describing diffusion in velocity space is used,

$$\left(\frac{\partial f}{\partial t}\right)_C = \frac{\partial}{\partial v}D_{\alpha\beta}\frac{\partial f}{\partial v}\,. \tag{11}$$

The perturbation field is determined by Maxwell equations

$$\nabla\times\mathbf{E} = \frac{i\omega}{c}\,\mathbf{B}\,, \tag{12}$$

$$\nabla\times\mathbf{B} = -\frac{i\omega}{c}\,\mathbf{E} + \frac{4\pi}{c}\,\mathbf{j}\,. \tag{13}$$

The kinetic model describes the current density

$$\mathbf{j} = e\int \mathrm{d}^3p\;\mathbf{v}\;f\,, \tag{14}$$

which is then used to compute MHD-moments and physical quantities according to Maxwell equations.

Profiles

In order to study the influence of different profiles on the instability, different TEXTOR profiles and additional analytic profiles are used.

Numerical adaptation

The solution of the eigenmode equation (6) is computed using numerical Runge-Kutta integration between pinch axis $r = 0$ and pinch wall $r = r_w$. The following boundary conditions are used,

$$[\xi_r, \xi_r'] = [1,1]\;;\; r = 0\,, \tag{15}$$

$$[\xi_r, \xi_r'] = [0,1]\;;\; r = r_w\,. \tag{16}$$

From each boundary an integration is performed and the two results are matched at an arbitrary point r_A. From the continuity of ξ_r and ξ_r' at r_A follows the system for (ξ_r, ξ_r'),

$$C_1\xi_1 + C_2\xi_2 = 0\,,$$

$$C_1\xi_1' + C_2\xi_2' = 0\,. \tag{17}$$

$C_1, C_2 = const.$, ξ_1' radial derivative.

The frequency γ for which the determinant of system (15) becomes zero corresponds to the eigenfrequency of the instability. The eigenfunction ξ for the MHD-instability is therefore a superposition of the two fundamental solutions ξ_1, ξ_2.

$$\xi = C_1\xi_1 + C_2\xi_2 \tag{18}$$

is plotted as $(r\xi)$ against r.

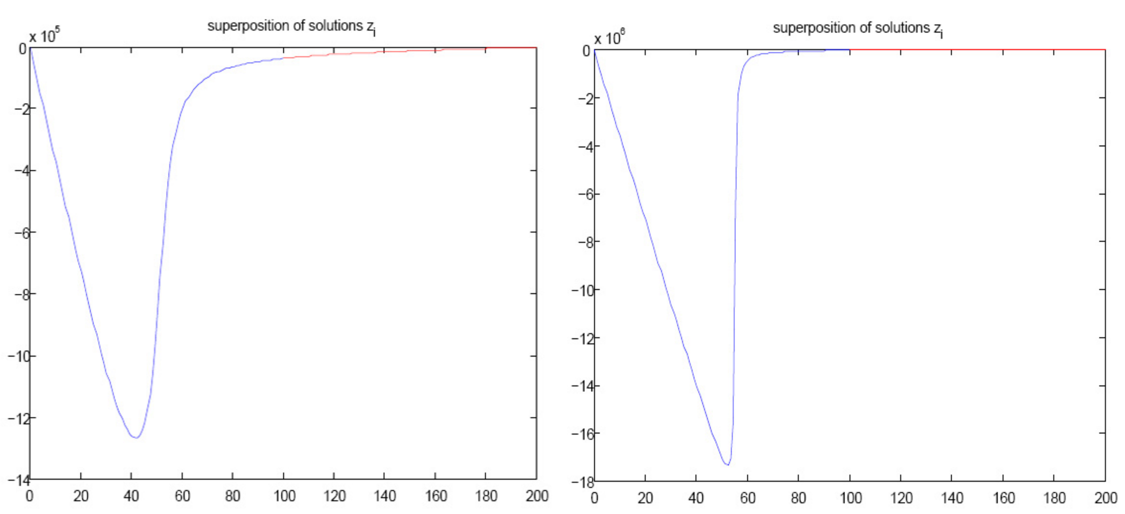

Figure 1: Eigenfunctions $(r\xi_r)$ vs. r for analytic profiles from MHD model, $k = -2\cdot10^{-3}$, $(m,n) = (1,-1)$, $r_w = 200cm$. Left: $\gamma = 2.038\cdot10^5\;s^{-1}$. Right: Lowered ∇p, and drop in eigenfrequency to $\gamma = 1.135\cdot10^5\;s^{-1}$

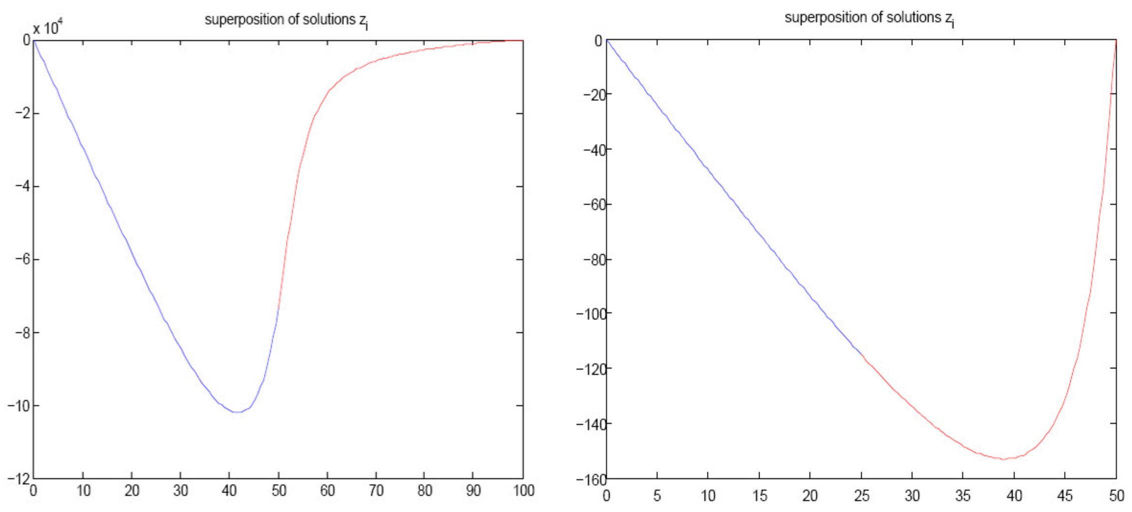

Figure 2: Eigenfunctions $(r\xi_r)$ vs. r with changes of wall position r_w for analytic profiles with $k = -2\cdot10^{-3}$, $(m,n) = (1,-1)$ and ∇p is the same for both cases. Left:$r_w = 100cm$, $\gamma = 2.028\cdot10^5\;s^{-1}$. Right: Lowered $r_w = 50cm$, and drop in eigenfrequency to $\gamma = 1.245\cdot10^5\;s^{-1}$

It was possible to reconstruct expected behaviour of instabilities with the MHD model. Fig(1) shows the result for $(m,n) = (1,-1)$ where a drop of the eigenfrequency is found, if the pressure gradient is decreased. For further decrease of p and ∇p the traced instability disappears completely, what is equal to a stabilisation due to pressure decrease.

Fig(2) shows the tracing of the same instability for unchanged p and ∇p, if the wall radius of the pinch is lowered. From initially $r_w = 200cm$ the eigenfrequency drops for $r_w = 100cm$ and even more for $r_w = 50cm$ and disappears completely at $r_w \approx 40cm$.

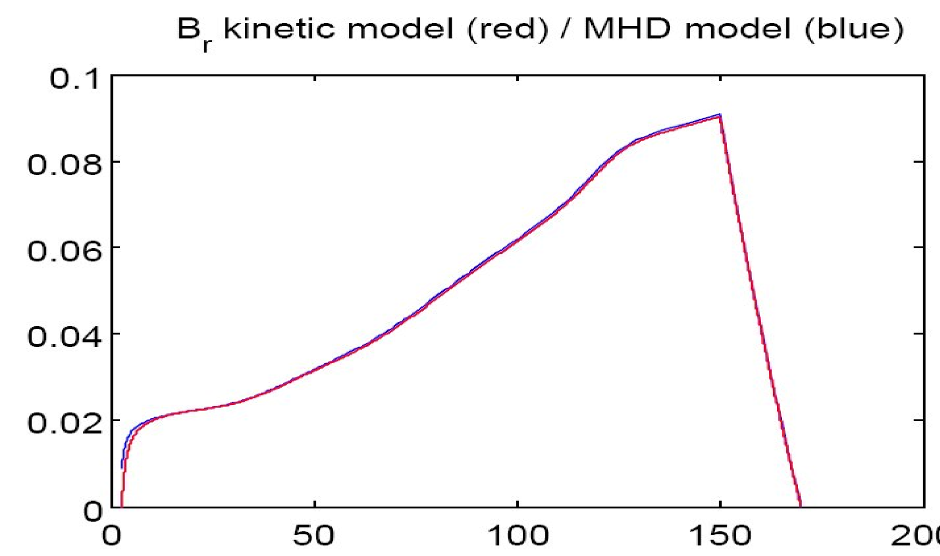

Figure 3: Comparison for B_r vs. r from kinetic model and MHD model for analytic profiles and frequency $\gamma = 1.4413\cdot10^5\;s^{-1}$ with $k = -2\cdot10^{-3}$, $(m,n) = (1,-1)$.

Currently several comparisons and further instability research by the kinetic code are in performance. Fig(3) shows a comparison of the B_r result between the kinetic and the MHD model inside the pinch. For the shown case both match each other quite good.

Conclusions

- The MHD model has been checked to reproduce the known trends with changes.
- The electromagnetic field computed from the kinetic model stays in good agreement with MHD results. This provides a basis to apply the kinetic model for cases where MHD theory is violated.

Acknowledgements

This work, supported by the European Communities under the contract of Association between EURATOM and the Austrian Academy of Sciences, was carried out within the framework of the European Fusion Development Agreement. The views and opinions expressed herein do not necessarily reflect those of the European Commission. Additional funding is provided by the Austrian Science Foundation, FWF, under contract number P19889-N16.

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