

# GOCE Gradients for Regional Gravity Field Solutions with Least Squares Collocation

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## Introduction

The GOCE mission is dedicated to determine the global static part of the gravity field. Its gradiometer instrument provides, for the first time in history, gravity gradient observations from space. This measurement type offers a new opportunity to derive improved gravity field solutions.

At regional scale, Least Squares Collocation (LSC) is a standard method for deriving the Earth's gravity field. However, the use of gravity gradients in LSC as in-situ observations is not straightforward. Important aspects concerning the filtering of raw gradient observations and the issue of frame transformations are outlined in this study.

Finally, geoid solutions from real gradient data and from a combination of gradient data and simulated terrestrial data are presented.

## Data

- GOCE gradients: Level-1b, November 2009, sampling 5s (Fig. 1), signal < d/o 50 removed
- Reference gravity model: EGM2008
- Gravity anomalies: simulated from EGM2008 on a 1/4° grid, d/o 50–250, white noise ( $\sigma = 3\text{mGal}$ ) added
- Investigated area: 75°–68° W, 14°–25° N
- Signal variances  $k_n$ : degree variances from EGM2008

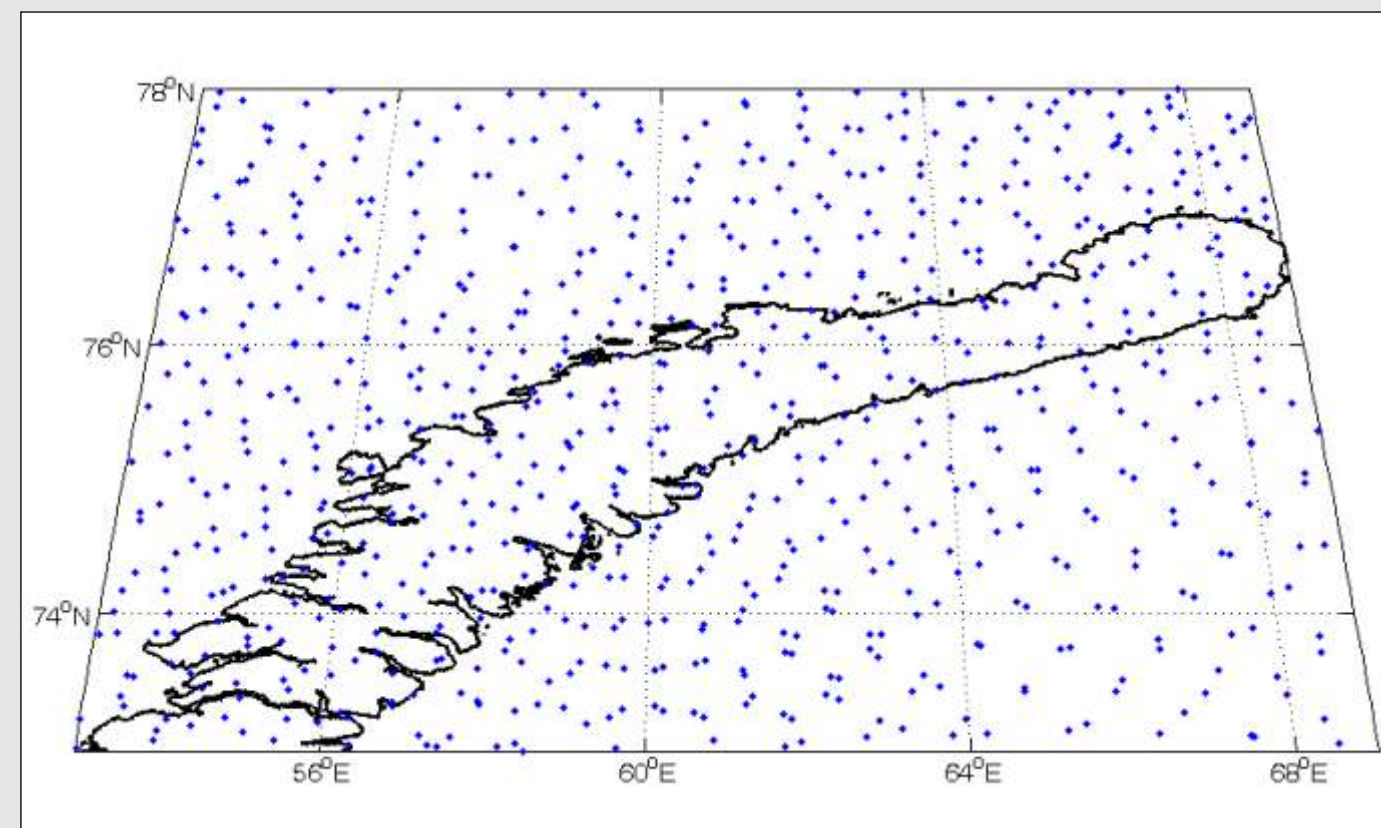


Fig. 1: Test area and data distribution of GOCE gradients for November 2009

## Results

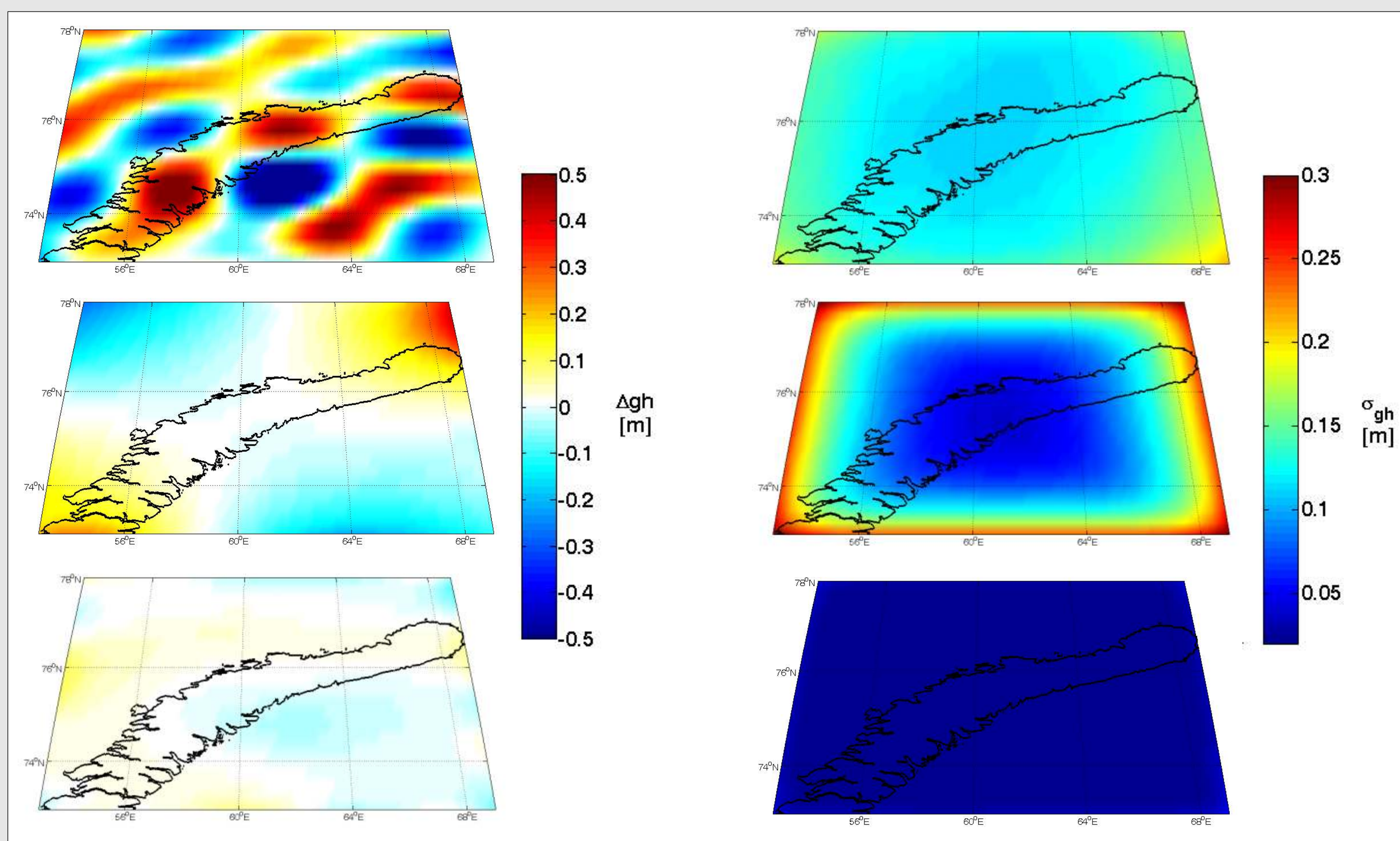


Fig. 6: Geoid height differences to EGM2008 (left) and standard deviations (right) in [m] using GOCE  $V_{xx}$ ,  $V_{yy}$ ,  $V_{zz}$  gradients (top), gravity anomalies (middle) and combined data (bottom).

- Geoid heights can be derived by solely GOCE gradients with an accuracy of  $\pm 12$  cm; differences w.r.t EGM2008 are in the order of 50 cm showing high-frequency patterns (Fig. 6, top)
- Using gravity anomalies, the accuracy is in the order of  $\pm 4$  cm; long-wavelength differences occur (Fig. 6, middle)
- The combination of gradients and gravity anomalies (Fig. 6, bottom) improves the solution with maximum standard deviations of  $\pm 2$  cm; differences w.r.t EGM2008 are also in the order of a few centimeters

## Methods

### Least Squares Collocation

- Prediction of gravity field signals  $s$  from arbitrary other gravity quantities  $l$   
$$s = C_{sl}(C_{ll} + C_{nn})^{-1}l$$
- Requirement: signals are a functional of the disturbing potential
- Relation via statistical correlation (covariances  $C_{ll}$ ,  $C_{sl}$ ) using a covariance function of the form

$$C(P, Q) = \sum_{n=2}^N k_n \frac{R^n}{r r'} P_n(\cos \theta) P_n(\cos \theta')$$

### Filtering Gradients

- Observations are measured in the gradiometer reference frame (GRF) and are superimposed with coloured noise
- Assumption: uncorrelated gradient tensor components

### Wiener filtering

- Requires noise-free reference gradient signal  $x$  and information on gradiometer error behaviour (PSD of noise  $n$ )

$$W = F[W] = \frac{S_x}{S_x + S_n} \dots \text{filter in spectral domain}$$

- Output: filtered gradient signal  $l$  (Fig. 2) and error covariance function from filter error  $C_{ee}$  (Fig. 3) for subsequent set-up of noise covariances  $C_{nn}$

$$l = F^{-1}[W F_l] \quad S_e = S_l - W S_l \quad C_{ee} = F^{-1}[S_e]$$

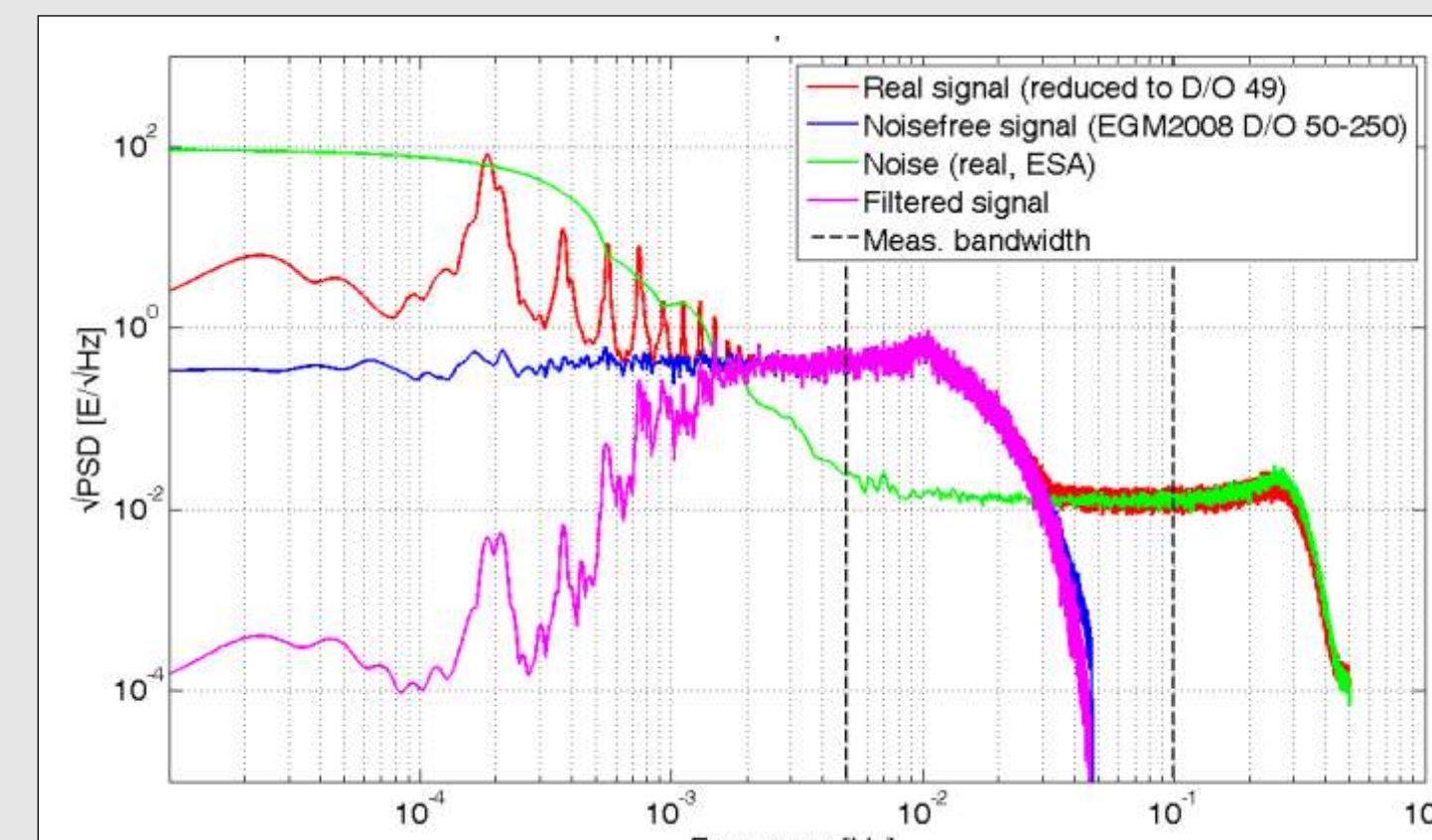


Fig. 2: PSD of GOCE  $V_{zz}$  gradients in GRF from d/o 50 to 250: real GOCE data (red), simulated from EGM2008 (blue), noise from ESA (green) and filtered (magenta)

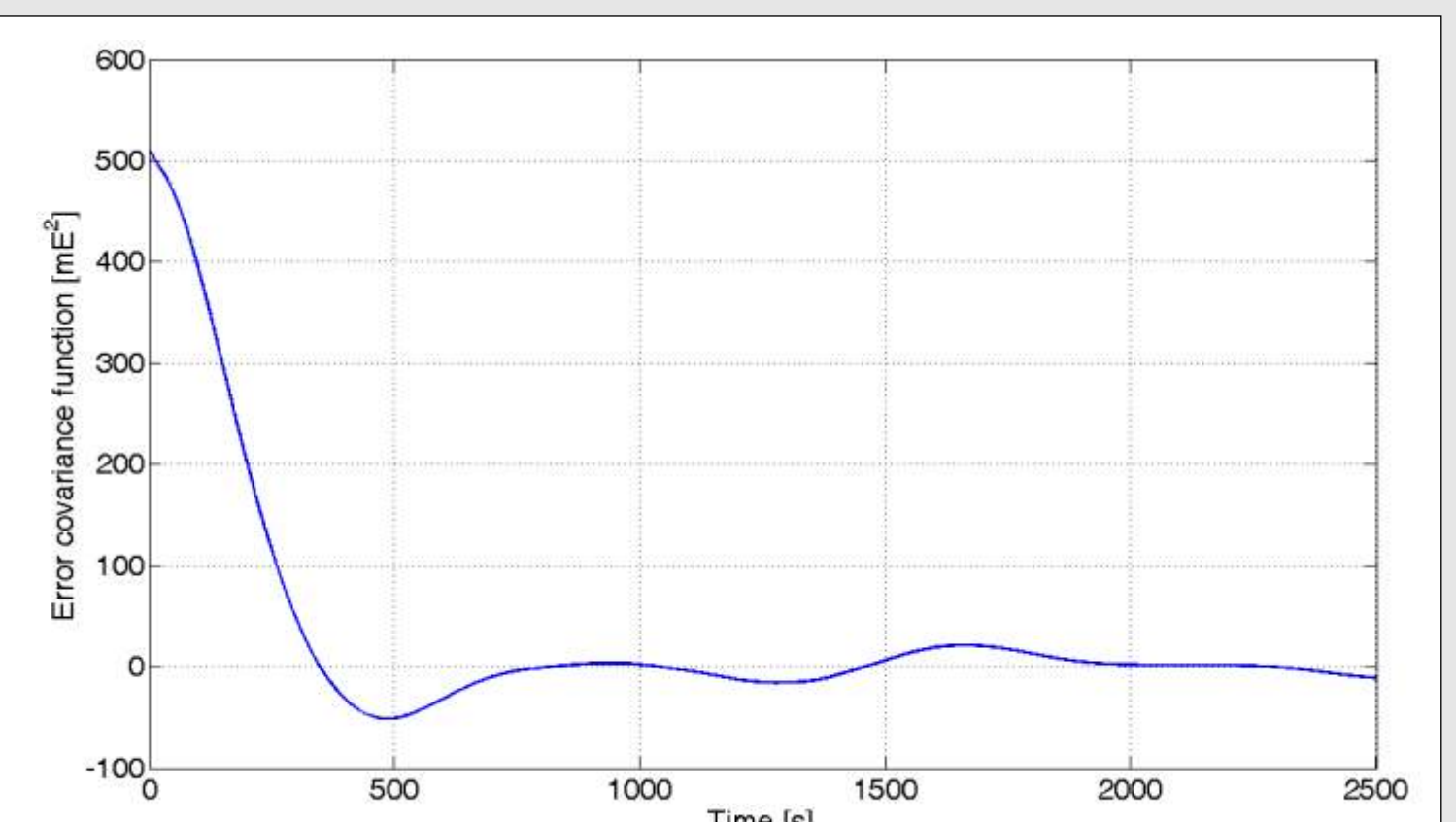


Fig. 3: Auto-covariance function of the filter error applying Wiener filtering to GOCE  $V_{zz}$  gradient time series

### Frame Transformations

- There are 3 different reference frames being of concern for this study:
  - Gradients are observed in the Gradiometer Reference Frame (GRF).
  - Collocation takes place in Local North Oriented Frame (LNOF).
  - Wiener filtering should be performed in Local Orbit Reference Frame (LORF).

Transformations of the gradient tensor must be avoided due to weak determined off-diagonal components (Fig. 4).

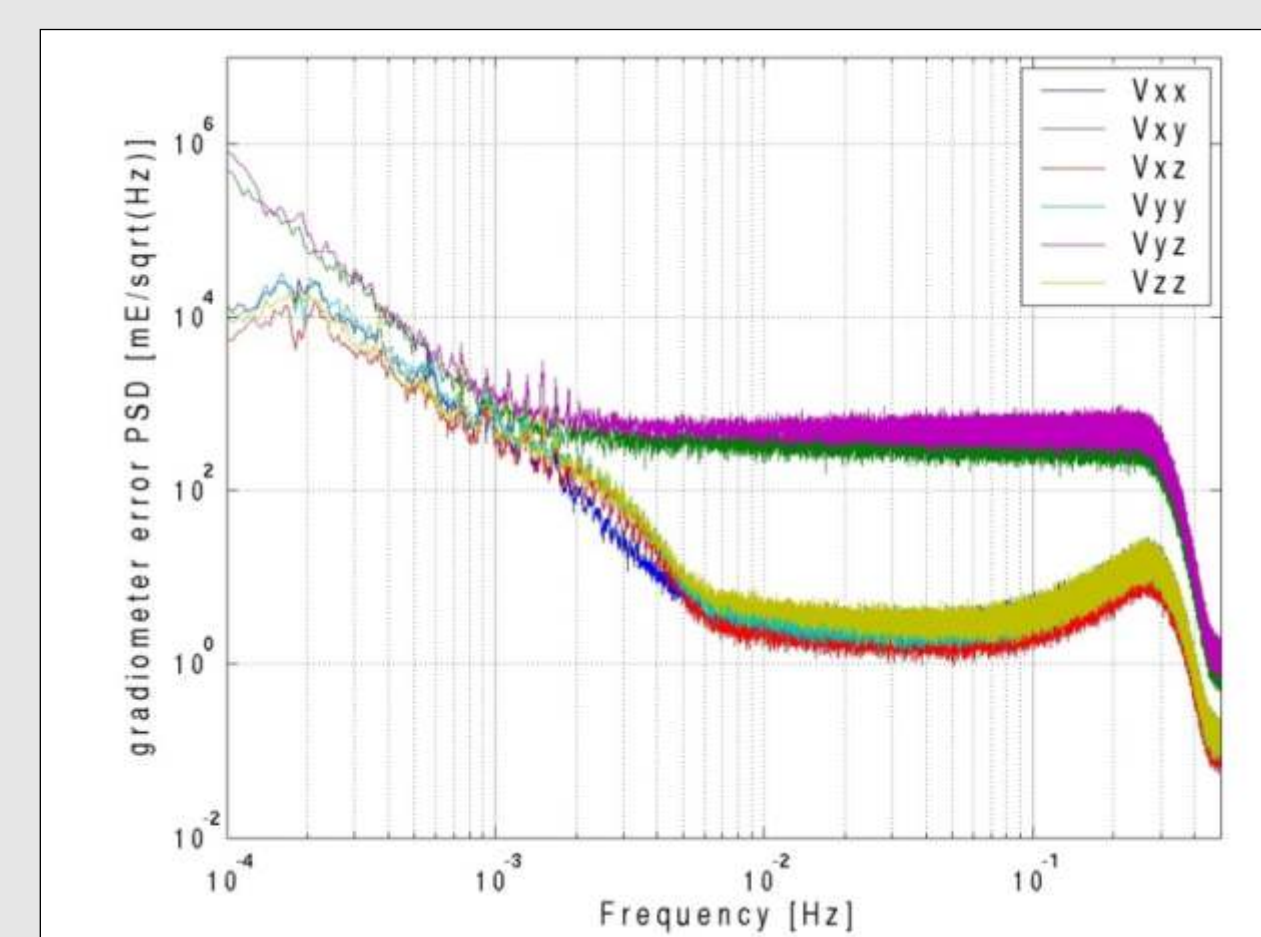


Fig. 4: Gradiometer noise PSD

### Strategy (Fig. 5):

- Gradient tensor  $V$  in GRF
- filtering  $V$  in GRF
- rotation of  $C_{ll}$  to GRF
- set-up of  $C_{nn}$  in GRF

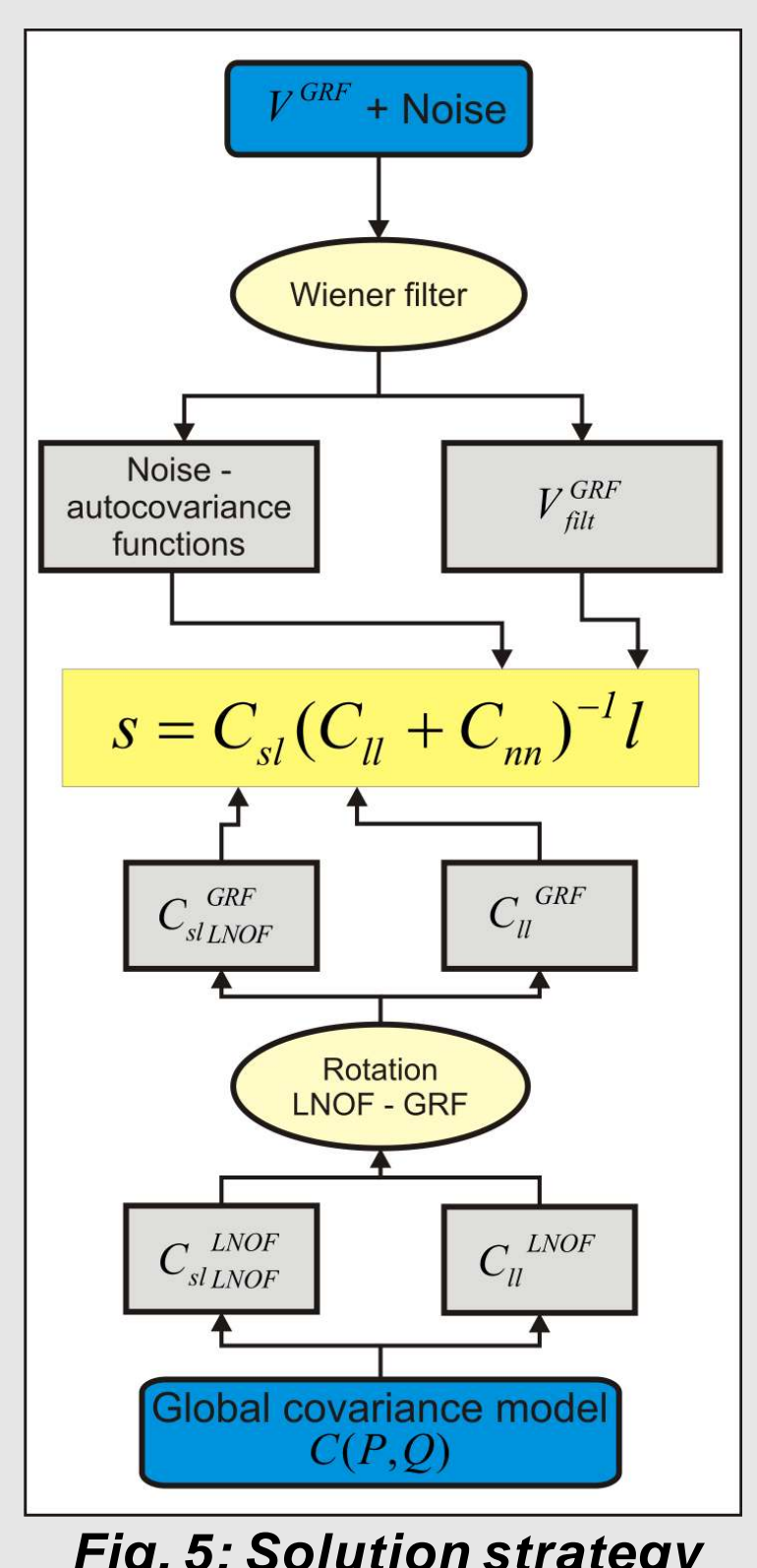


Fig. 5: Solution strategy

## Conclusions

- GOCE gradients contribute significantly to regional geoid determination in the medium wavelengths of the gravity signal.
- The combination with terrestrial data is necessary to stabilize the downward continuation of the gradients.
- Despite neglecting the requirement of filtering gradient data in LORF, the Wiener filter results show a good performance.
- A proper covariance function valid for each data type has still to be found.

## References

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