PAMM · Proc. Appl. Math. Mech. **4**, 520–521 (2004) / **DOI** 10.1002/pamm.200410241

Sound Insulation of Porous Plates

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Because of the still increasing noise pollution the numerical simulation of acoustic problems becomes more and more important. One essential aspect is the numerical treatment of noise insulation of solid walls. The main noise source is the bending vibration of separating components. In general, they consist of porous material, e.g., concrete or bricks. To take into account the porous structure as well as the damping effect of the porosity of these components a poroelastic plate theory is developed. The Finite Element implementation of this plate theory shows the importance of taking porous materials into account.

1 Poroelastic Mindlin Plate Theory

The combination of a poroelastic constitutive assumption and a plate theory was published by Theodorakopoulos and Beskos [1]. They used the Kirchhoff theory assuming thin plates and neglected any in-plane motion to obtain purely bending vibrations. Here, a poroelastic theory of Biot is used and a Mindlin plate is considered to model the bending vibrations of moderately thick poroelastic plates.

Following Biot's approach [2] to model the behavior of porous media, an elastic skeleton with a statistical distribution of interconnected pores saturated by a fluid is considered. The poroelastic plate theory is developed, starting from the formulation of the internal energy of a poroelastic 3D-continuum

$$E = \frac{1}{2}\int\limits_{\Omega}(\sigma_{ij}\epsilon_{ij} + p\zeta)d\Omega \quad \longrightarrow \quad \delta E = \frac{1}{2}\int\limits_{\Omega}(\sigma_{ij}\delta\epsilon_{ij} + \delta p\zeta)d\Omega \tag{1}$$

using the total stress $\sigma_{ij} = \sigma_{ij}^s + \sigma^f\delta_{ij}$, the pore pressure p, and the linear strain $\epsilon = \frac{1}{2}(u_{i,j} + u_{j,i})$ with the solid displacement u_i. For this poroelastic continuum the constitutive equations read

$$\sigma_{ij} = G(u_{i,j} + u_{j,i}) + \frac{2G\nu}{1-2\nu}u_{k,k}\delta_{ij} - \alpha\delta_{ij}p \qquad \zeta = \alpha u_{k,k} + \frac{\phi^2}{R}p \qquad i,j,k = 1,2,3 \tag{2}$$

with the shear modulus of the solid frame G, the Poisson's ratio ν, and Biot's effective stress coefficient α. Additionally to the total stress σ_{ij}, as a second constitutive equation, the variation of fluid volume per unit reference volume ζ is introduced with the material constant R and the porosity ϕ.

Further, the fluid transport in the interstitial space is modeled with a generalized Darcy's law

$$q_i = -\kappa(p_{,i} + \rho_f\omega^2 u_i + \frac{\rho_a + \phi\rho_f}{\phi}\omega^2 v_i) \tag{3}$$

where κ denotes the permeability, v_i the relative fluid to solid displacement, and q_i the specific flux. The density of the solid and the fluid is denoted by ρ_s and ρ_f, where the bulk density can be calculated from $\rho = \rho_s(1-\phi) + \phi\rho_f$. To describe the interaction between fluid and skeleton the apparent mass densityρ_a was introduced by Biot [2]. Using the above equations the principle of virtual work for this continuum can be established

$$\begin{aligned} &\int\limits_{\Omega}\left[\left(G(u_{i,j} + u_{j,i}) + \left(\frac{2G\nu}{1-2\nu}u_{k,k} - \alpha p\right)\delta_{ij}\right)\delta u_{i,j} + (\alpha u_{i,i} + \frac{\phi^2}{R}p)\delta p\right]d\Omega \\ &\quad = \int\limits_{\Gamma}\left[(t_i\delta u_i - \frac{1}{i\omega}q_i n_i\delta p\right]d\Gamma + \int\limits_{\Omega}\left[\left[\beta p_{,i} + \omega^2(\rho - \beta\rho_f)u_i + F_i\right]\delta u_i + \left[\frac{\beta}{\omega^2\rho_f}p_{,i} - \beta u_i\right]\delta p_{,i}\right]d\Omega\ . \end{aligned} \tag{4}$$

Now, the usual kinematic plate assumptions proposed by Mindlin have to be inserted

$$u_\alpha(x_1,x_2,x_3) = x_3\psi_\alpha(x_1,x_2) \qquad \alpha,\beta = 1,2 \qquad u_3(x_1,x_2,x_3) = w(x_1,x_2) \tag{5}$$

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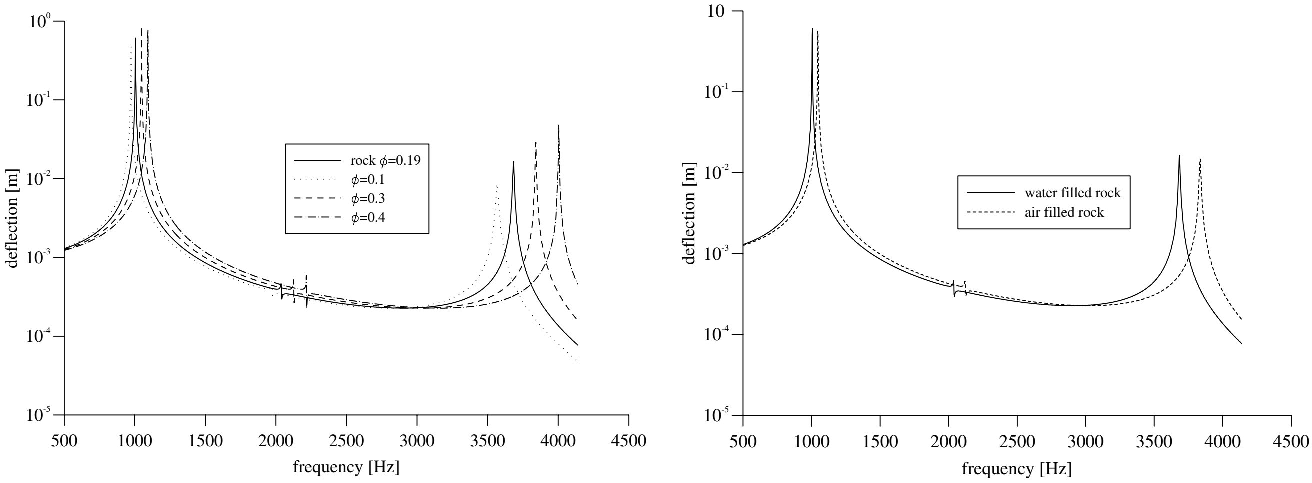

Fig. 1 Deflection at point P: Variation of the porosity ϕ of a rock plate and Comparison of rock plates with water and air filled pores

with the frequency dependent rotations ψ_α and the frequency dependent deflection w. It is assumed that $\sigma_{33} \ll \sigma_{11}, \sigma_{22}$ holds yielding $\sigma_{33} \approx 0$, which is used to eliminate the derivative of the out-of-plane displacement u_3 with respect to the x_3-direction

$$\sigma_{33} = 0 \quad \longrightarrow \quad u_{3,3} = \frac{\alpha(1-2\nu)}{2G(1-\nu)} - \frac{\nu}{1-\nu}\, x_3 \psi_{\alpha,\alpha} \; . \tag{6}$$

Applying (5) and (6) on the variational principle (4), the variational principle for the poroelastic Mindlin plate theory is achieved. From this the equations for w, ψ_α, and $Q = \int_{-h/2}^{+h/2} x_3 p(x_1, x_2, x_3) dx_3$ are obtained

$$G(\psi_{\alpha,\alpha} + w_{,\alpha\alpha})h_s + \beta \Delta p + \int_{-\frac{h}{2}}^{\frac{h}{2}} F_3 x_3 dx_3 + \omega^2(\rho - \beta\rho_f)hw = 0 \tag{7}$$

$$\frac{h^3}{12} G \psi_{\alpha,\beta\beta} + \frac{h^3}{12} G \frac{1+\nu}{1-\nu} \psi_{\beta,\alpha\beta} - (\alpha \frac{1-2\nu}{1-\nu} - \beta) Q_{,\alpha} + h_s G(\psi_\alpha + w_{,\alpha}) + \omega^2(\rho - \beta\rho_f)\psi_\alpha \frac{h^3}{12} = 0 \tag{8}$$

$$\frac{\beta}{i\omega\rho_f} Q_{,\alpha\alpha} - i\omega \left[\frac{\phi^2}{R} + (\alpha - \beta) \frac{\alpha(1-2\nu)}{2G(1-\nu)} \right] Q - i\omega(\alpha - \beta) \frac{1-2\nu}{1-\nu} \frac{h^3}{12} \psi_{\alpha,\alpha} + \frac{\beta}{i\omega\rho_f} \left[\frac{h}{2} \Delta q - \Delta p \right] = 0 \; . \tag{9}$$

with $\Delta p = p_o - p_u, \Delta q = q_o - q_u$, and $h_s = 5/6h$. Choosing ansatz functions for the unknowns w, ψ_α, and Q in this principle of virtual work results in a Finite Element formulation. Usually in poroelastic FEM the pore pressure is approximated with an ansatz order less than the deflection and rotation. Here, for first testing, the same ansatz order is applied.

2 Examples

As a test example a hinged plate of the dimensions $0.2m \times 0.2m \times 0.01m$ with an uniformly distributed dynamic load is implemented in a FEM-program. The plate is discretized using 256 linear elements. The material data are those of a rock (Berea sandstone), $G = 6 \cdot 10^9 \frac{N}{m^2}, \nu = 0.33, \varrho = 2458 \frac{kg}{m^3}, R = 4.7 \cdot 10^8 \frac{N}{m^2}, \varrho_f = 1000 \frac{kg}{m^3}, \alpha = 0.778, \kappa = 1.9 \cdot 10^{-10} \frac{m^4}{Ns}$. In figure 1, the deflection at point P ($0.125m \times 0.1m$) is plotted versus the frequency to study the influence of the porosity and of the interstitial fluid. The first figure shows a shift of the three visible eigenfrequencies. For smaller values of the porosity ($\phi = 0.1$) the eigenfrequencies become smaller and for larger values of the porosity ($\phi = 0.3, \phi = 0.4$) they are shifted to higher frequencies. In the second figure, a rock plate with water filled pores is compared to a rock plate with air filled pores. A dependence of the eigenfrequencies from the interstitial fluid can be recognized. For the air filled plate the eigenfrequencies are shifted to higher frequencies.

References

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