Thin Layer Transition Matrix description applied to the Finite Element Method

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***Abstract*— The behavior of an electromagnetic wave impinging on a thin layer of arbitrary material can be described with a vector circuit interpretation of a transition matrix model. This paper follows the idea of applying such a matrix to the finite element method (FEM). Especially, several very thin layers composed to an electrically not so thin layer may be treated comfortable. The method will be compared firstly on simple dielectric layer with a known analytic solution. In consequence the example of a thick layer modeled by a number of thin layers will be given, as well.**

I. Introduction

High attention is paid nowadays on reflection and absorption of electromagnetic radiated power for defining standards in human ambience as well as in electromagnetic compatibility. Effective shielding and coating of materials are in demand. For analyzing thin layers must be treated, as well. When applying FEM, the modeling of several very thin layers becomes numerically cumbersome. Out of several posibilities, eg. making use of frequency selective surfaces [1], we incorporated interface conditions in the FEM, obtained by a transition matrix model.

Beginning with the Maxwell equations

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}, \qquad \nabla \times \vec{H} = j\omega\epsilon\vec{E}, \tag{1}$$

the fields as well as the the nabla operator may be split into tangential components (subscript t) and its normals (subscript n):

$$\vec{E} = \vec{E}_t + \vec{n}\,E_n,\ \vec{H} = \vec{H}_t + \vec{n}\,H_n,\ \nabla = \nabla_t + \frac{\partial}{\partial n}\vec{n}. \tag{2}$$

The normal components may be substituted [2] and after performing some algebra the relations for the tangential field components become

$$\frac{\partial \vec{E}_t}{\partial n} = (j\omega\mu\bar{I}_t + \frac{j}{\omega\epsilon}\nabla_t\nabla_t)\cdot(\vec{n}\times\vec{H}_t) \tag{3}$$

$$\frac{\partial(\vec{n}\times\vec{H}_t)}{\partial n} = (j\omega\epsilon\bar{I}_t + \frac{j}{\omega\mu}\vec{n}\times\nabla_t\vec{n}\times\nabla_t)\cdot\vec{E}_t. \tag{4}$$

Now, according to the idea in [2], $\vec{E}_t$ and $\vec{n}\times\vec{H}_t$ in (3,4) may be interpreted like voltage and current in a circuit transition matrix model.

$$\left\{ \begin{array}{c} \vec{E}_{t+} \\ \vec{n}\times\vec{H}_{t+} \end{array} \right\} = \left[\begin{array}{cc} \bar{a}_{11} & \bar{a}_{12} \\ \bar{a}_{21} & \bar{a}_{22} \end{array} \right] \cdot \left\{ \begin{array}{c} \vec{E}_{t-} \\ \vec{n}\times\vec{H}_{t-} \end{array} \right\} \tag{5}$$

In case of a number of N composed layers, the transition matrixes of all layers have to be multiplied to end up with a final transition matrix $[A]$.

$$[A] = \prod_{i=1}^{N} [A]_i. \tag{6}$$

To succeed in expressing the tangential magnetic fields, the admittance matrix $[Y]$ from (6) may be derived:

$$\left\{ \begin{array}{c} \vec{n}\times\vec{H}_{t+} \\ \vec{n}\times\vec{H}_{t-} \end{array} \right\} = \left[\begin{array}{cc} \bar{Y}_{11} & \bar{Y}_{12} \\ \bar{Y}_{21} & \bar{Y}_{22} \end{array} \right] \cdot \left\{ \begin{array}{c} \vec{E}_{t+} \\ \vec{E}_{t-} \end{array} \right\} \tag{7}$$

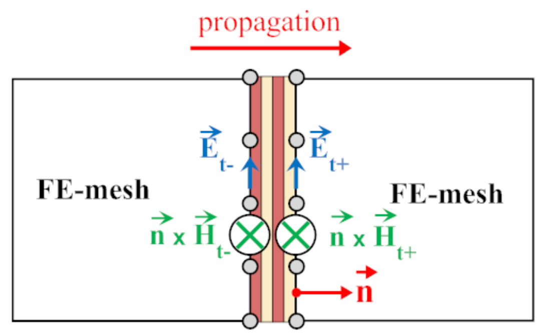

Fig. 1. Used notation, wave propagation indicated, layers not FE-meshed.

While incorporating the conditions given in (7) by evaluating a surface integral of $\vec{n}\times\vec{H}_t$ the FE-meshes adjoining the multi-layer will be linked together without meshing the layers itself. In our case this has been done in the weak form of the Galerkin equations for the well known $\vec{A}, v$-formulation [3]. For the mesh truncation at the outer boundaries of the FE-model perfectly matched layers have been applied, as suggested in [4].

II. Conclusions

The treatment of thin layer field problems with a standard finite element method has been presented. To avoid the meshing of very thin layers a way to link the domains adjoined to the layers has been found while incorporating a surface integral in the FE-formulation. Therefore, the layer characteristics have been derived with the aid of transition matrixes and the admittance matrix. So, a very convenient procedure has been presented which later on may be extended for meta-materials and periodic structured crystal applications.

References

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