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Effect of the anomalous transport on the interaction of resonant magnetic field perturbations with tokamak plasmas

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Motivation: Fluid modeling of stochastic pedestal does not match experimental data at DIII-D

Heat transport in DIII-D during ELM mitigation by means of resonant magnetic field perturbations (RMPs) from the I-coil has been modeled with the help of 3D Monte Carlo fluid code E3D in Ref. [I.Joseph, et al, Nucl.Fusion **48** 045009 (2008)]. The MHD heat transport equations are solved assuming that DIII-D magnetic field is a superposition of the equilibrium magnetic field $\mathbf{B}_0$ and perturbation magnetic field $\tilde{\mathbf{B}}$ taken for the vacuum case. It has been observed that heat transport is significantly over-estimated by this model as compared to the experiment.

There are at least two reasons for this discrepancy:

1) Violation of the MHD approximation in the transport model,

2) Significant effect of shielding of the RMPs by the plasma.

Estimation of the quasi-linear heat transport coefficients for the kinetic regime has shown that these coefficients still lead to an over-estimation of heat transport when taking the perturbation field in vacuum approximation. Therefore, plasma shielding effect on RMPs must be significant.

Motivation: Drift & kinetic effects should be included

In linear approximation, plasma shielding of RMPs has been studied in detail within the drift MHD theory, see e.g. [A.Cole and R.Fitzpatrick, Phys.Plasmas **13** 032503 (2006)].

Nevertheless, for the estimation of this effect in DIII-D, linear kinetic plasma response model has been used in [Heyn et al, Nucl.Fusion **48** 024005 (2008)] because in collisionless plasma MHD approximation may be violated for the following reasons:

1) Landau damping effect which is absent in MHD approximation can be important if perturbation frequency ω_E in the rotating reference frame where radial electric field is zero, is larger than collision frequency ν. For electrons, ω_E and ν_e can be comparable in DIII-D while for ions ω_E usually exceeds ν_i.
2) The width of the resonant layer must be much larger than Larmor radius. This is true for electrons but not for ions.

Linear kinetic plasma response model

In [Heyn et al, 2008] the set of Maxwell equations for the electromagnetic field with harmonic time dependence, $\tilde{\mathbf{E}}, \tilde{\mathbf{B}} \propto \exp(-i\omega t)$,

$$\nabla \times \tilde{\mathbf{E}} = \frac{i\omega}{c}\tilde{\mathbf{B}}, \qquad \nabla \times \tilde{\mathbf{B}} = -\frac{i\omega}{c}\tilde{\mathbf{E}} + \frac{4\pi}{c}\tilde{\mathbf{j}},$$

has been solved for the simplified tokamak geometry in the form of a straight radially inhomogeneous periodic plasma cylinder with the rotational transform of the magnetic field. Plasma response current $\tilde{\mathbf{j}}$ has been computed with help of kinetic theory using the Hamiltonian action-angle variables $(\boldsymbol{\theta}, \mathbf{J})$ as

$$\tilde{\mathbf{j}}(\mathbf{r}) = e \int \mathrm{d}^3\theta \int \mathrm{d}^3 J \; \mathbf{v}_c(\boldsymbol{\theta}, \mathbf{J}) \; \tilde{f}(\boldsymbol{\theta}, \mathbf{J}) \; \delta(\mathbf{r} - \mathbf{r}_c(\boldsymbol{\theta}, \mathbf{J})),$$

where angles $\theta^\alpha = (\phi, \theta, z)$ are gyrophase, poloidal angle and z coordinate, resp., and actions $J_\alpha = (J_\perp = m_0 v_\perp^2/(2\omega_c), p_\theta, p_z)$ are perpendicular adiabatic invariant, θ and z components of the canonical momentum, resp.

Kinetic plasma response model with Krook collision term

In [Heyn et al, 2008] $\tilde{f}$ satisfies the linearized kinetic equation with Krook collision term,

$$(-i\omega + \nu)\,\tilde{f} + \Omega^\alpha \frac{\partial \tilde{f}}{\partial \theta^\alpha} = -e\left(\tilde{\mathbf{E}} + \frac{1}{c}\mathbf{v}_c \times \tilde{\mathbf{B}}\right) \cdot \frac{\partial \mathbf{r}_c}{\partial \theta^\alpha}\frac{\partial f_0}{\partial J_\alpha},$$

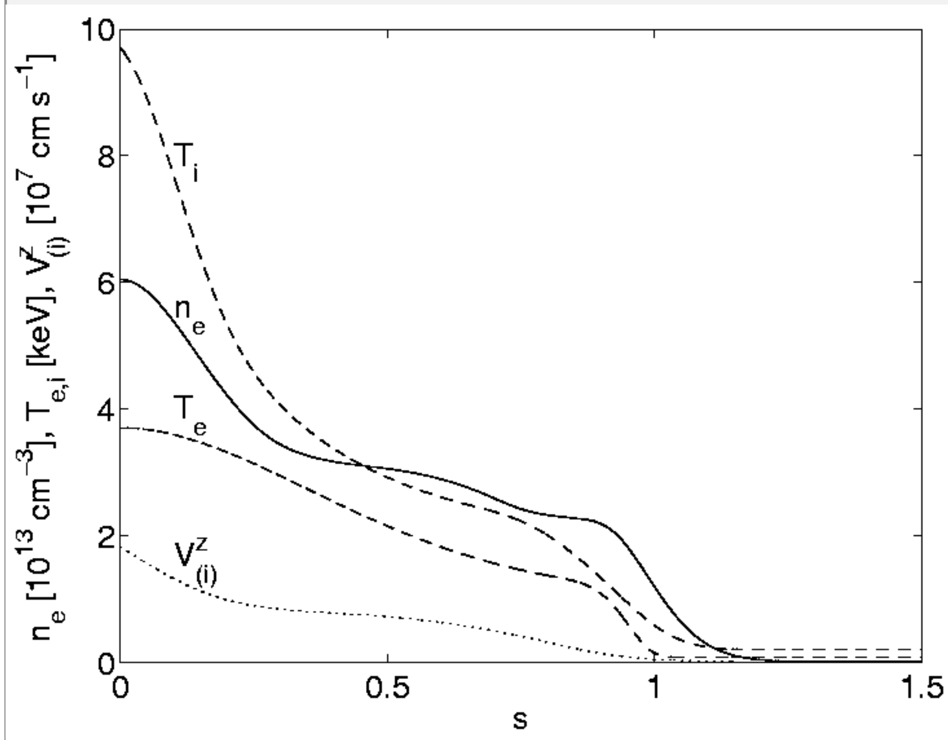
where $\Omega_\alpha = (\omega_c, v_\parallel B_0^\vartheta / B_0, v_\parallel B_0^z / B_0)$ are canonical frequencies and f_0 is the equilibrium distribution function. With the help of Fourier analysis over θ and z variables and expansion over Larmor radius, integral expression for the current $\tilde{\mathbf{j}}$ had been simplified to a differential operator acting on $\tilde{\mathbf{E}}$. Thus, Maxwell equations had been reduced to a set of ordinary differential equations with respect to the radius r which had been solved numerically. The result for a particular Fourier mode, $\mathbf{E}, \mathbf{B} \propto \exp(im\vartheta + in\varphi)$ where $\varphi = z/R$, can be presented in terms of a "form factor"

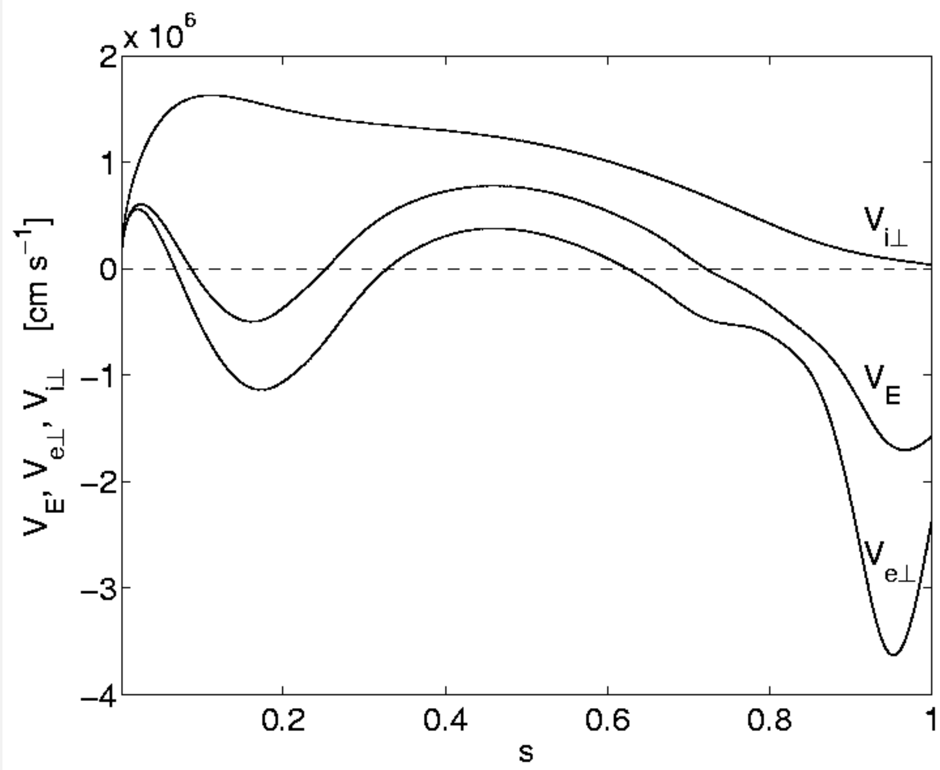
$$T_{m,n}(s) = B_r^{(plas)}(r)/B_r^{(vac)}(r),$$

being the ratio of radial magnetic field in plasma to the field in vacuum. Here $s \approx (r/a)^2$ is a normalized toroidal flux.

Equilibrium plasma parameters show importance of drift effects

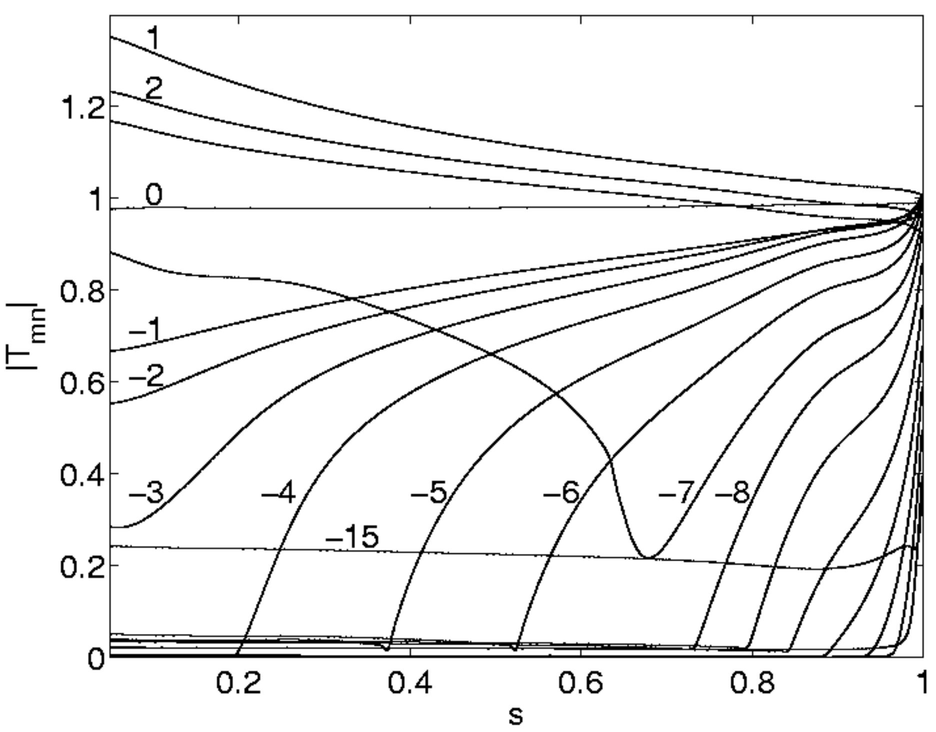
Input: $q(s)$, $n_e(s)$, $T_e(s)$, $T_i(s)$, $V_i^{tor}(s)$ corresponding to the DIII-D shot 126006. They determine electron and ion fluid velocities $V_{e\perp}(s)$ and $V_{i\perp}(s)$ and electric drift velocity $V_E(s)$. Here $s = \psi_{\rm tor}/\psi_{\rm tor,edge}$ is the normalized toroidal flux.

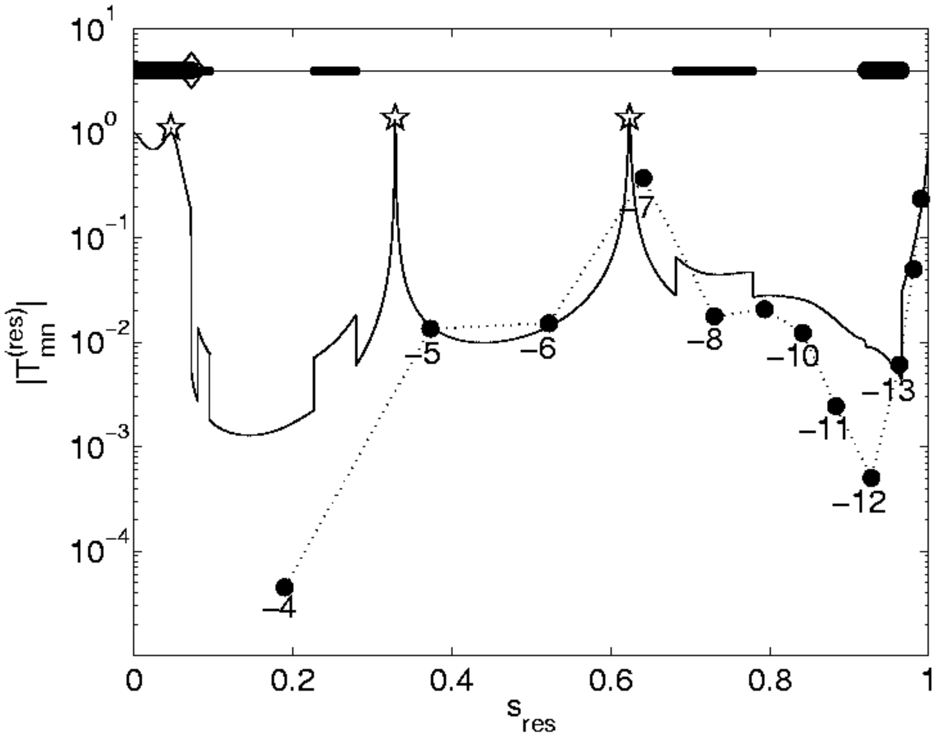




Screening seen in both kinetic and drift MHD models

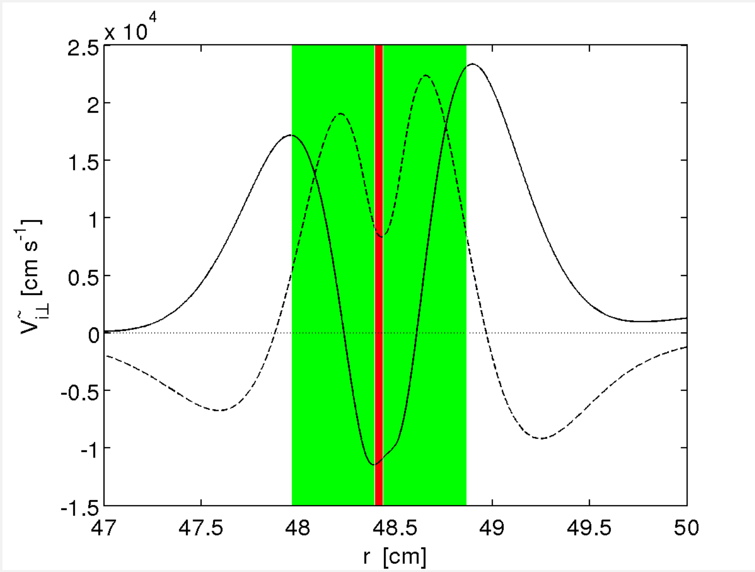
Left: $T_{m,n}(s)$ for the kinetic model for $n = 3$. Poloidal wavenumbers m label the curves.
Right: $T_{m,n}$ values at respective resonant surfaces s_{res}. Points - kinetic model (m values - below), solid line - drift MHD model assuming that m changes continuously. Stars mark zeros of the perpendicular equilibrium electron fluid velocity, $V_{e\perp} = 0$, where field penetration is strongest.





The kinetic vs. drift tearing layers

1) Both kinetic and drift MHD model predict strong shielding of RMPs by the plasma in the core and relatively small shielding near the separatrix.
2) Results of the kinetic model with Krook collision term qualitatively agree with the results of drift MHD model despite the facts that ion Landau damping dominates ion collisional damping and ion Larmor radius is comparable with the width of the resonant zone:



(Here the perturbed ion fluid velocity, $\tilde{V}_{i\perp}$, computed within the kinetic model, is plotted together with the MHD resonance zone width shown in red and with Larmor radius scale shown in green.) This agreement between the kinetic and the MHD models is due to the domination of electrons which can still be described by the MHD.

Enhanced decorrelation due to anomalous transport

1) Quasilinear diffusion describing turbulent fluctuations of the electromagnetic field,

$$(-i\omega+\nu)\,\tilde{f}+\Omega^{\alpha}\frac{\partial\tilde{f}}{\partial\theta^{\alpha}}-\frac{\partial}{\partial J_{\alpha}}D_{A}^{\alpha\beta}\frac{\partial\tilde{f}}{\partial J_{\beta}}=-e\left(\tilde{\mathbf{E}}+\frac{1}{c}\mathbf{v}_{c}\times\tilde{\mathbf{B}}\right)\cdot\frac{\partial\mathbf{r}_{c}}{\partial\theta^{\alpha}}\frac{\partial f_{0}}{\partial J_{\alpha}}\equiv Q$$

was found to destroy the resonance condition more strongly than collisions. The magnitude of the resonant response current is determined by the Cherenkov resonance, $\omega-\omega_E-k_{\|}v_{\|}=0$. The precise condition is destroyed by Coulomb collisions, modifying $v_{\|}$, and by small radial transport displacements, modifying $k_{\|}$.

2) Random radial displacement during the time τ_d, $\delta r\sim\sqrt{D_{\perp}\tau_d}$, where $D_{\perp}$ is perpendicular diffusion coefficient, leads to the random change of the wave-particle phase of the order $v_{\|}\delta k_{\|}\tau_d=v_{\|}k_{\|}^{\prime}\tau_d\delta r$. Phase correlation is lost if this change is $O(1)$ which gives the decorrelation time

$$1/\tau_d=\nu_{\mathrm{eff}}=D_{\perp}^{1/3}\left(v_{\|}k_{\|}^{\prime}\right)^{2/3}$$

Effect on tearing layer physics

Assuming the resonant perturbation in the form of a single Fourier mode, $Q \propto \exp(im\vartheta + ik_z z)$, and retaining only the spatial terms in the diffusion tensor $D_A^{\alpha\beta}$ for the gyro-average of this equation describing the resonant response current one has

$$i\left(k_\parallel v_\parallel - \omega - i\nu\right)\langle\tilde{f}\rangle_\phi - D_\perp \frac{\partial^2}{\partial r^2}\langle\tilde{f}\rangle_\phi = \langle Q\rangle_\phi,$$

Introducing the dimensionless radial variable x according to $\nu_{\text{eff}}\, x = k'_\parallel v_\parallel\,(r - r_{\text{res}}) - \omega$ where r_{res} is the resonant radius, kinetic equation is

$$\left(ix + \frac{\nu}{\nu_{\text{eff}}}\right)\langle\tilde{f}\rangle_\phi - \frac{\partial^2}{\partial x^2}\langle\tilde{f}\rangle_\phi = \frac{1}{\nu_{\text{eff}}}\langle Q\rangle_\phi.$$

If $\nu \gg \nu_{\text{eff}}$, maximum value of $\langle\tilde{f}\rangle_\phi$ at the resonance $x = 0$ is limited by the collision frequency ν, $\langle\tilde{f}\rangle_\phi \sim \nu^{-1}\langle Q\rangle_\phi$, however, in the case $\nu \ll \nu_{\text{eff}}$ this value is limited by the diffusion term so that $\langle\tilde{f}\rangle_\phi \sim \nu_{\text{eff}}^{-1}\langle Q\rangle_\phi$, i.e. ν_{eff} effectively replaces Coulomb collision frequency ν. For the typical DIII-D core plasma parameters ν_{eff} is more than an order of magnitude larger than ν. Similar relation holds also for ITER parameters.

Effect on mode locking threshold

Effects of the anomalous transport on the RMP shielding and mode locking thresholds can be estimated using the results of [R. Fitzpatrick, Phys,Plasmas **5** 3325 (1998)] for the resistive-inertial regime with the replacement $\nu \to \nu_{\text{eff}}$. Unlike ν, the effective "collision" frequency ν_{eff} does not scale with density n_0. As a result, effective parallel plasma conductivity scales with n_0 linearly which means stronger shielding of RMPs for higher plasma densities.

According to the torque balance [R. Fitzpatrick, 1998] mode locking threshold is determined by the ratio of the torque acting from the RMP on the plasma to the anomalous viscous force. In the resistive-inertial regime scaling of this ratio with density is weak, i.e. the threshold value of the RMP amplitude $\tilde{B}_{thr} \sim n_0^{1/4}$. Replacement of ν with ν_{eff} leads to a stronger scaling, $\tilde{B}_{thr} \sim n_0^{1/2}$.

Note that

$$\nu_{\text{eff}} \sim \left(k'_{||}\right)^{2/3} \sim n^{2/3} \left(\frac{1}{q}\frac{\mathrm{d}q}{\mathrm{d}r}\right)^{2/3}$$

where n is the toroidal wavenumber and q is a safety factor. Thus, near the separatrix where shear parameter strongly increases, shielding of RMPs must be strongly reduced compared to the predictions of the drift MHD theory (and kinetic theory without the account of the anomalous transport).

The RMP interaction with plasma can be treated within the linear (quasilinear) approximation to large island sizes

In the low frequency range the main non-linear effect of the RMP on the plasma is caused by the radial perturbation magnetic field $\tilde{B}_r$ which leads to radial transport and respective re-distribution of parallel plasma response current. During the decorrelation time τ_d, this field leads to the radial displacement of resonant particle by

$$\delta r_{\rm NL} = \tau_d v_{\|} \tilde{B}_r / B_0.$$

The nonlinear regime is achieved if the nonlinear wave-particle phase shift during the decorrelation time, $\tau_d k_{\|}' v_{\|} \delta r_{\rm NL}$, is greater than one. If $\delta r_{\rm NL}$ is smaller than the random displacement by anomalous diffusion, δr, nonlinear phase shift is negligible. Expressed in terms of the separatrix half-width, $\delta r_{\rm isl}$,

$$\delta r_{\rm NL}/\delta r = \delta r_{\rm isl}^2/(16 \delta r^2), \qquad \delta r_{\rm isl} = 4 \left| \tilde{B}_r / (k_{\|}' B_0) \right|^{1/2}.$$

For typical DIII-D core plasma parameters and toroidal mode $n = 3$ one obtains that linear theory is applicable up to island sizes of the order of $\delta r_{\rm isl} \sim 1$ cm.