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NEW ALGORITHMS FOR THE SIMULATION OF THE SEQUENTIAL TUNNEL EXCAVATION WITH THE BOUNDARY ELEMENT METHOD (EURO:TUN 2007)

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Abstract. *The Boundary Element Method (BEM) is well suited for modelling problems in infinte domains. This situation is present in the case of sequential excavation in tunnelling.*
The traditional way for the modelling of a sequential excavation process is the use of the multiple region BEM. Within an infinite domain numerous finite subdomains are embedded. Each of the finite subdomains represents the rock volume to be excavated in an excavation step. Between two subsequent analysis steps, the problem geometry changes such, that one or more of the finite subdomains become inactive, i.e., are excavated. Stiffness matrices are calculated for each subdomain and assembled to a global system of equation. For each excavation step the whole equation system has to be solved, including unknowns at interface nodes belonging to regions of subsequent analysis steps.
In the proposed new algorithms the surface of the excavated part of the tunnel is discretized only. The loading for the following excavation step is determined by an internal result computation of the BEM. As it is well known, the determination of internal domain results with the BEM exactly satisfies the condition of equilibrium, this means no error is introduced by this procedure. Thus, a single region problem only has to be solved as a consequence, and the efficiency of the calculation will be increased in comparison to the method described above. The loading of a subsequent excavation step is determined either by displacement or stress calculation at internal points. From internal stresses tractions at the excavation surfaces can be determined. If displacement at internal points are evaluated intermediate calculations on finite regions are necessary. In this case previous calculated internal displacements are the Dirichlet boundary conditions on a single finite region. From these calculations the loading of the following excavation step is obtained.
The results of a sequential tunnel excavation in 2D and 3D for the proposed new algorithms are presented and a comparison with the traditional method mentioned above and to Finite Element calculations is done.

1 INTRODUCTION

The New Austrian Tunnelling Method is characterised by a complicated sequence of excavations and installation of support systems. In this context the modelling of the sequential tunnel excavation in an elastic continuum will be discussed in this paper. The numerical method which will be applied is the Boundary Element Method (BEM).

The BEM is well suited for modelling problems in infinte domains. This situation is present in the case of sequential excavation in tunnelling. The traditional way for the modelling of the sequential excavation process with the BEM is the use of multiple regions[1]. The technique of multiple regions is shown to be applied successfully for the modelling of the sequential excavation process[2, 3]. For the purpose of introduction and recapitulation of modelling the sequential excavation in tunneling this method will be explained first.

Then two computational methods will be explained which are rather new. The main idea is to apply a single region boundary value problem for each excavation step instead of using the concept of multiple regions. The problem in this context is the determination of the excavation loads of the current load step.

One possiblity is the evaluation of displacements at interiour points which belong to the excavation surface of the subsequent excavation steps. With these displacements intermediate calculations on a single region have to be performed to get the solution for the loading. Another possibility is the evaluation of stresses at interiour points situated at the excavation surface. The tractions at the excavation surface are calculated easily by a transformation of the stresses obtained before. This two methods will be explained in detail and results are shown for an example in 2D.

2 EXCAVATION PROCESS MODELLED BY THE MULTIPLE REGION CONCEPT

In the following sections first the theory of multiple regions in the BEM is explained briefly and then an example in 3D is presented.

2.1 Theory of the Multiple Region Boundary Element Method (MRBEM)

The displacement boundary integral equation in its discretized form is given by [1].

$$\mathbf{c} \cdot \mathbf{u}\left(P_i\right) = \sum_{e=1}^{E} \sum_{n=1}^{N} \Delta \mathbf{U}_{ni}^{e} \cdot \mathbf{t}_{n}^{e} - \sum_{e=1}^{E} \sum_{n=1}^{N} \Delta \mathbf{T}_{ni}^{e} \cdot \mathbf{u}_{n}^{e} \tag{1}$$

$\Delta\mathbf{U}_{ni}^{e}$ and $\Delta\mathbf{T}_{ni}^{e}$ are the integrated kernel shape function products of the element *e* for a collocation point P_i. $\mathbf{u}_{n}^{e}$ and $\mathbf{t}_{n}^{e}$ are the displacements and the tractions respectively for the element *e* at elementnode *n*. Writing Eq. 1 for all collocation points leads to a system of equations of the form:

$$\mathbf{T} \cdot \mathbf{u} = \mathbf{U} \cdot \mathbf{t} \tag{2}$$

where $\mathbf{u}$, $\mathbf{t}$ are vectors containing all displacements and tractions at the boundary, respectively, and $\mathbf{T}$, $\mathbf{U}$ are coefficient matrices assembled by gathering all element contributions. If a mixed boundary value problem is given, i.e. $\mathbf{u}$ is prescribed on some portion of the boundary and $\mathbf{t}$ on the remaining part, on both sides of Eq. 2 knowns and unknowns are involved. Thus, Eq. 2 has to be rearranged to the form

$$\mathbf{B} \cdot \mathbf{x} = \mathbf{f} \tag{3}$$

with all unknown values placed on the left hand side and all known values on the right hand side.

In the case of sequential excavation, which has to consider the staged advance of the excavation, a special solution method is needed. At the interfaces between regions, both the displacements and tractions are unknown. Therefore the number of unknowns is increased and additional equations are necessary. These equations can be obtained from the conditions of equilibrium and compatibility at the region interfaces.

Various methods to solve multiple region problems have been proposed(see [5, 6, 7, 8, 9]). In this work the so called stiffness matrix assembly, developed by Beer [1], is used. This approach is similar to the approach taken by the finite element method. A stiffness matrix is computed for each region. The coefficients of this matrices are the nodal interface tractions due to unit displacements. The stiffness matrices of all regions are assembled in the same way as in the FEM. This method is also suitable for coupling boundary elements to finite elements.

A multiregion problem may be fully or partially coupled. In the first case all nodes of a system are treated as coupled. In the second case there is a differentiation between nodes which are coupled to other regions and nodes which are free nodes. In both methods the stiffness matrix is calculated for coupled nodes. In the fully coupled method all nodes of the system are treated as coupled nodes and the size of the assembled stiffness matrix nearly remains the same during the steps of excavation. In the partially coupled method a change from coupled nodes to free nodes is involved by removing regions throughout the excavation process. The size of the assembled stiffness matrix reduces every step of excavation. The partially coupled method is preferred to the fully coupled method because of the reduction of the size and the effort for solving the equation system. Consider for example a tunnel excavation which is performed in

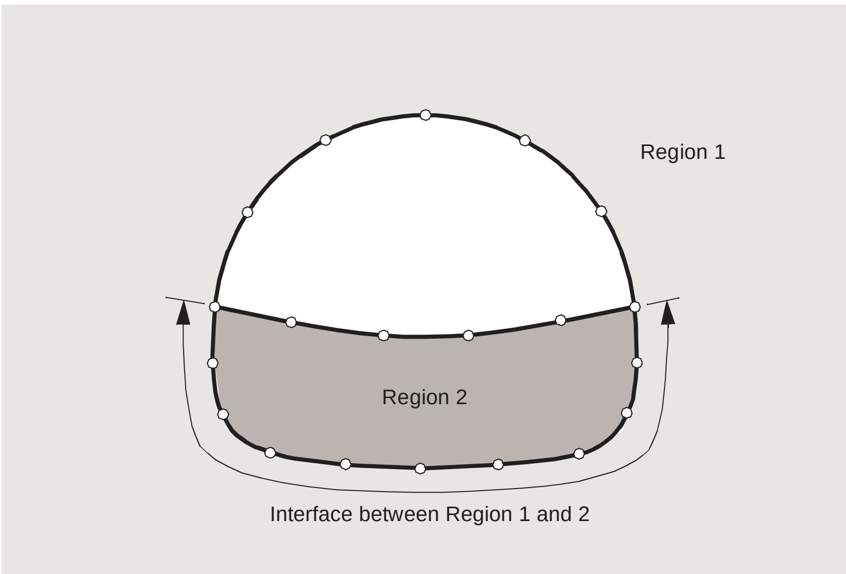

Figure 1: Partially coupled problem

top heading and bench excavation, which is shown in Fig. 1. The actual load case consists of a Neumann boundary condition which is specified at the surface of the removed top heading material as a result of the initial stress state.

As can be seen in Fig. 1 only some of the nodes of region 1 are connected to region 2.

Obviously it would be more efficient to consider only the interface nodes for the calculation of the stiffness matrix , i.e. only those nodes that are belonging to more than one region. The nodes which are not connected to another region (free nodes) are assigned the given boundary conditions, in this case tractions at the free surface (see Fig. 2).

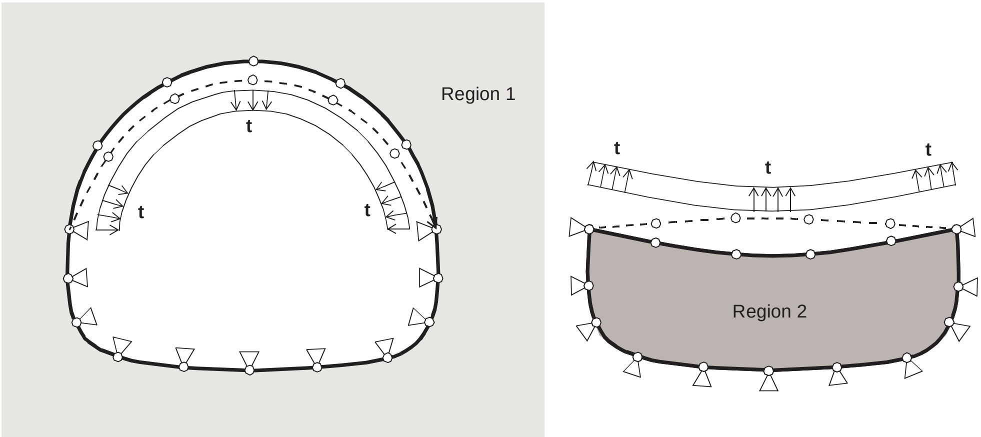

Figure 2: Fixed interface nodes and given boundary conditions at free nodes

For region 1 and 2 the following equations can be written:

$$\mathbf{B}^N \left\{ \begin{array}{c} \mathbf{t}_{c0}^N \\ \mathbf{u}_{f0}^N \end{array} \right\} = \mathbf{f}_0^N \tag{4}$$

$$\mathbf{B}^N \left\{ \begin{array}{c} \mathbf{t}_{cn}^N \\ \mathbf{u}_{fn}^N \end{array} \right\} = \mathbf{f}_n^N \qquad n = 1, 2 \ldots Ndofc \tag{5}$$

where N is the region number, $\mathbf{B}^N$ is the assembled left-hand side. $\mathbf{f}_0^N$ in Eq. 4 is the right-hand side obtained from the given boundary conditions, i.e. the tractions at the free nodes due to excavation loading and zero values of displacements at the coupled nodes. The unknown vectors in Eq. 4 contain tractions at the coupled nodes $\mathbf{t}_{c0}^N$ and displacements at the free nodes $\mathbf{u}_{f0}^N$, respectively.

The same equation is shown in Eq. 5, but here the loading consists of unit displacements. Unit displacements are applied at every interface node for every degree of freedom ($n = 1, 2 \ldots Ndofc$). The solution vectors of tractions $\mathbf{t}_{cn}^N$, belonging to the interface, are assembled to the stiffness matrix $\mathbf{K}^N$ and the vectors of displacements at the free nodes $\mathbf{u}_{fn}^N$ to the matrix $\mathbf{A}^N$ (shown in Eq. 7).

The final solution for $\mathbf{t}_c^N$, the tractions at the interface nodes, and $\mathbf{u}_f^N$, the displacements at the free nodes, can be expressed in terms of $\mathbf{u}_c^N$, the displacements at coupled nodes, by superposition

$$\left\{ \begin{array}{c} \mathbf{t}_c^N \\ \mathbf{u}_f^N \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{t}_{c0}^N \\ \mathbf{u}_{f0}^N \end{array} \right\} + \left\{ \begin{array}{c} \mathbf{K}^N \\ \mathbf{A}^N \end{array} \right\} \mathbf{u}_c^N \tag{6}$$

where the matrices $\mathbf{K}^N$ and $\mathbf{A}^N$ for region N are defined by:

$$\begin{aligned} \mathbf{K}^N &= [\mathbf{t}_{c1},\ \mathbf{t}_{c2},\ \ldots \mathbf{t}_{cNdofc}] \\ \mathbf{A}^N &= [\mathbf{u}_{f1},\ \mathbf{u}_{f2},\ \ldots \mathbf{u}_{fNdofc}] \end{aligned} \tag{7}$$

Using the conditions for equilibrium and compatibility at the interface, which are

$$\begin{aligned} \mathbf{t}_c^I + \mathbf{t}_c^{II} &= 0 \\ \mathbf{u}_c^I &= \mathbf{u}_c^{II} \end{aligned} \tag{8}$$

the stiffness matrices $\mathbf{K}^N$ and the vectors $\mathbf{t}_{c0}^N$ of all regions can be assembled into a global system of equations, which can be solved for the unknown interface displacements $\mathbf{u}_c$.

$$\mathbf{t}_{c0} + \mathbf{K} \cdot \mathbf{u}_c = \mathbf{0} \tag{9}$$

$\mathbf{K}$ is the assembled stiffness matrix and $\mathbf{t}_{c0}$ is the assembled vector of tractions at the interface nodes due to given boundary conditions at the free nodes. The remaining unknowns, displacements at the free nodes and tractions at the coupled nodes can be evaluated returning back from system to region level, by using Eq. 6.

2.2 Example of a tunnel excavation in 3D

In the following example the sequential excavation of a tunnel is analyzed with the multiple region boundary element approach. The results from this calculation are compared with the results from a finite element calculation. The finite element mesh can be seen in Fig. 3 (left). The mesh consists of about 10000 quadratic brick elements.

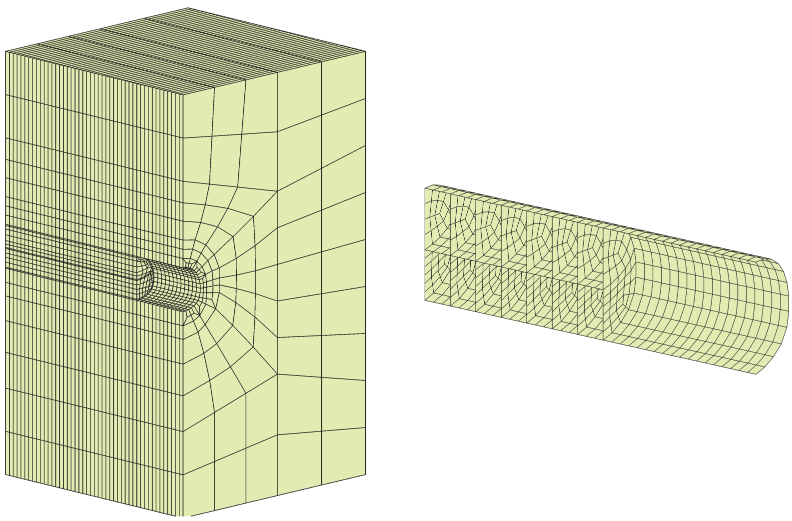

Figure 3: Finite element mesh (left) - boundary element mesh (right)

The staged advance of the tunnel is assumed as a full face excavation. The material parameters are the following:

Young's modulus:	$E = 5000\, MN/m^2$	Poisson's ratio:	$\nu = 0.3$
Specific weight:	$\gamma = 0.025\, MN/m^3$	Lateral pressure:	$k_0 = 0.5$

The virgin stress field is linearly distributed, whereas on the ground surface the stress is zero and in a depth of 400.0 m the virgin stress is:

$$\begin{array}{lll} \sigma_{xx} = -5.00\, MN/m^2 & \sigma_{yy} = -5.00\, MN/m^2 & \sigma_{zz} = -10.00\, MN/m^2 \\ \tau_{xy} = \quad 0.00\, MN/m^2 & \tau_{yz} = \quad 0.00\, MN/m^2 & \tau_{zx} = \quad 0.00\, MN/m^2 \end{array} \tag{10}$$

In Fig. 3 the boundary element mesh (right) can be seen. Linear boundary elements are used. The BE-mesh consists of 2418 elements and 27 regions (1 infinite region and 26 finite regions). The cross section of the tunnel is divided in top heading and bench. Therefore 13 regions of top heading and 13 regions of benches exist. In the following calculation at each load case a region of top heading and a region of bench is excavated simultaneously. A detailed comparison of BEM and FEM results of vertical displacements at the crown of the tunnel for each loadstep is shown in Fig. 4. It can be seen that the BEM-results and the FEM-results agree very well and the shape of the displacement curves of the BEM calculation compared with the FEM calculation is almost the same for each load case.

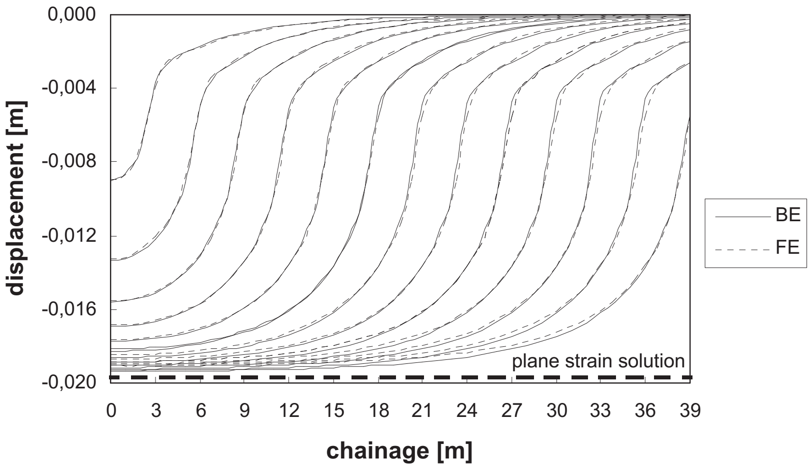

Figure 4: z-displacements at the crown of the tunnel for 13 load cases

3 NEW MODELLING STRATEGIES OF EXCAVATION

The method discussed before requires a predefinition of the complete geometry of the tunnel excavation problem. From beginning the calculation has to deal with all the regions, interfaces, etc., which in subsequent load cases will be part of the excavation process. This means, that the size of the equation system is determined by the number of all degrees of freedom at the entire geometry and it nearly remains the same for every load step of calculation.
The idea now is to use only a single boundary element region, representing the actual excavation surface. Displacements and stresses are calculated at points inside the domain, provided that the boundary conditions are known. For the respective load step excavation loads are determined with this approach. In the following sections two modelling strategies of sequential excavation are discussed.

3.1 New modelling strategy - internal displacement approach

The algorithm is explained best for an example in 2D. In Fig. 5 the geometry of a tunnel is shown in a longitudinal section. The excavation sequence is shown and the excavated parts of each step are indicated by hatched areas. First 5 steps of top heading excavation are modelled, followed by the bench excavations. In sum 10 load steps are calculated. Modelling a 2D excavation in such a way means an excavation of infinite extent out of plane, which of course is not a real tunnel excavation, but this model is appropriate for the demonstration of the algorithm.
With this method it is necessary to discretize the excavated tunnel surface only. A single bound-

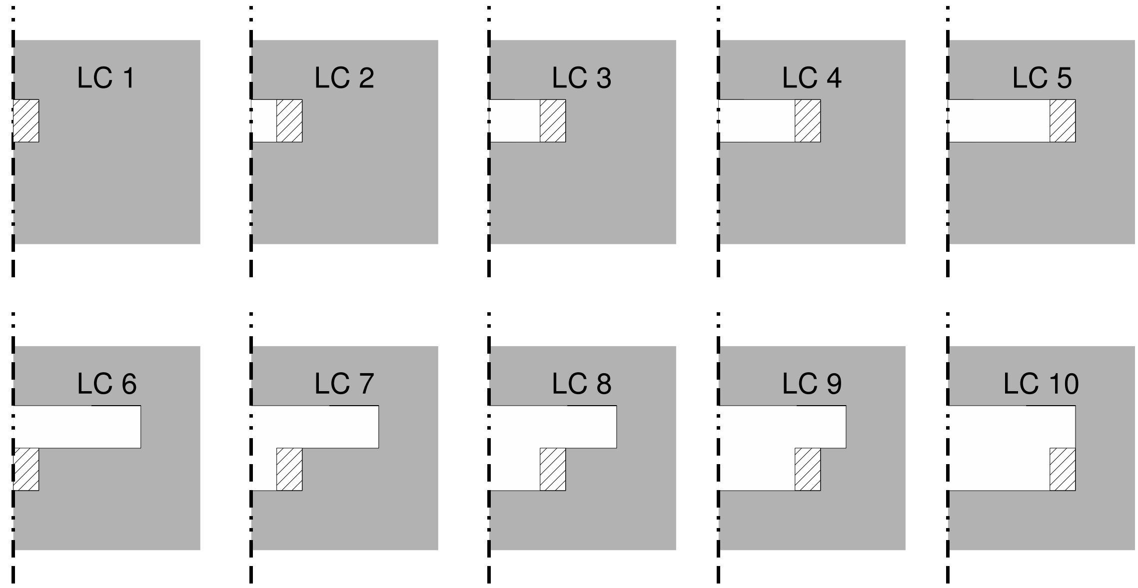

Figure 5: Longitudinal tunnel section - sequence of all load cases

ary element region is sufficient for the discretization of the respective load step which can be seen in Fig. 5.

The crucial part is the determination of excavation tractions for the current load case. This is explained next for load case LC4. In Fig. 6 the excavation steps from LC1 until LC4 are shown. The region to be excavated in LC4 (indicated by hatched areas) is shown in the sketches for each of the previous loadcases. The resultant displacements and tractions for LC4 are the sum of incremental results of all previous load cases. This implies that all the previous load cases have to be considered for the evaluation of the loading for LC4.

For the evaluation of excavation tractions internal displacements are calculated on nodes be-

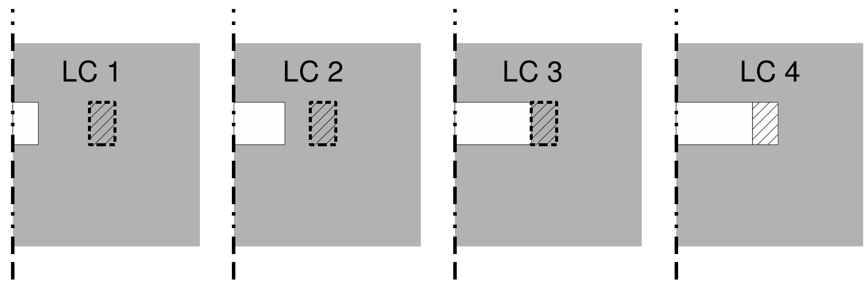

Figure 6: Excavation of LC4

longing to the region to be excavated. In the BEM Somigliana's identity is used for the computation of displacements in the interiour domain of a region. In Eq. 11 Somigliana's Identity is shown in its discretized form.

$$\mathbf{u}\left(P_i\right) = \sum_{e=1}^{E}\sum_{n=1}^{N} \Delta\mathbf{U}_{ni}^{e} \cdot \mathbf{t}_{n}^{e} - \sum_{e=1}^{E}\sum_{n=1}^{N} \Delta\mathbf{T}_{ni}^{e} \cdot \mathbf{u}_{n}^{e} \tag{11}$$

$\mathbf{u}_n^e$ and $\mathbf{t}_n^e$ are the known values of displacements and tractions at the boundary, respectively, $\Delta\mathbf{U}_{ni}^e$ and $\Delta\mathbf{T}_{ni}^e$ are the integrated kernel shape function products for node n at element e.

For LC1 and LC2 all the nodes are internal nodes, but for LC3 only a part of them. Thus, based on the finite region to be excavated (shown in Fig. 6 by hatched areas) two intermediate calculations have to be done dependent on the prevailing boundary conditions. This is shown in Fig. 7. For LC1 and LC2 internal displacements are calculated for all nodes of the finite region and they are the Dirichlet boundary conditions of the intermediate calculation. For LC3 not all nodes of the finite region are internal nodes and the boundary conditions are mixed (Dirichlet and Neumann). Fig. 7 shows the given boundary conditions and the solution values for the two intermdediate calculations.

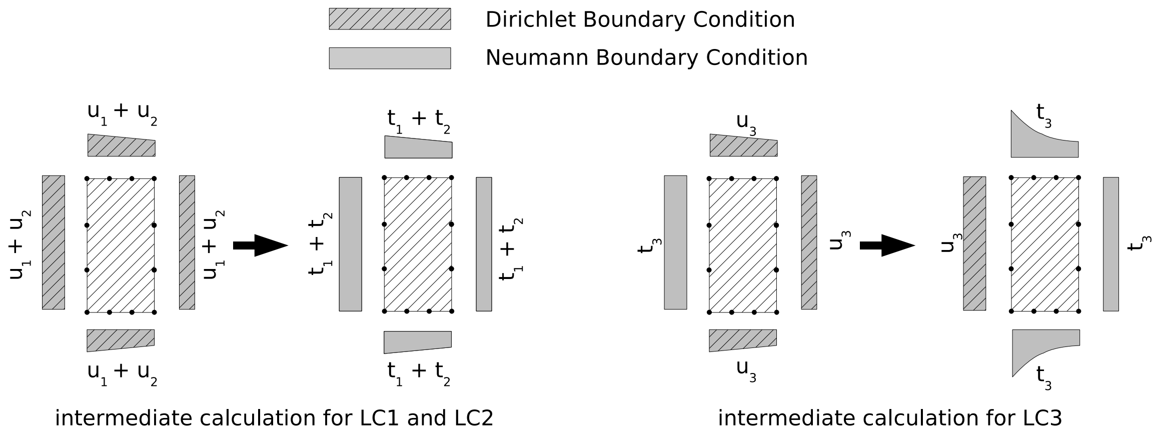

Figure 7: Intermediate calculation for LC1, LC2 and LC3

The resultend excavation tractions for LC4 are the sum of tractions of the two intermediate calculations completed by the tractions due to the virgin stress field. These excavation tractions are applied at the freed elements of the single region for LC4 (shown in Fig. 5 and Fig. 6). Now all boundary conditions for a single region boundary element calculation are known and the solution for the current load step is achieved by using Eq. 3.

3.2 Results - internal displacement approach

For the example shown in Fig. 5 a virgin stress field is assumed, with a horizontal stress and a vertical stress of constant value of $-5.00\,MN/m^2$. The Young's modulus is $E = 5000\,MN/m^2$ and the Poisson's ratio is $\nu = 0.3$. The size of each finite region is $3m$ in length and $5m$ in higth. Results of vertical displacements along the crown of the tunnel are shown in Fig. 8 for each load case. These results (LC?_NEW) are compared with the reference solution (LC_REF) and as can be seen they agree well.

3.3 New modelling strategy - internal stress approach

Compared to the method explained before the excavation tractions for this approach are calculated with internal stress evalutation. The stress at an internal point is calculated similar as

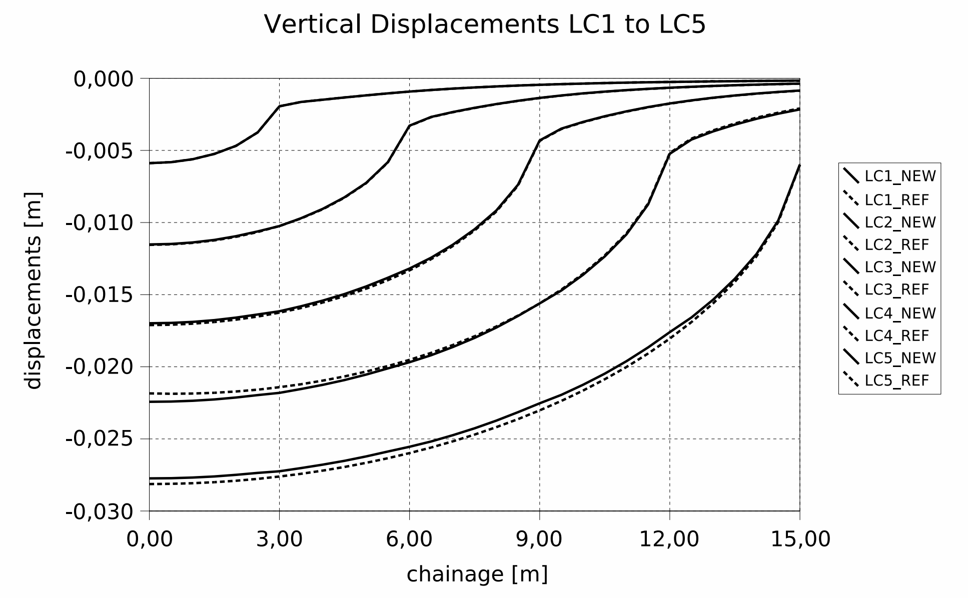

Figure 8: Vertical displacements at tunnel crown

displacements. Somigliana's Identity in the formulation of stress is shown in Eq. 12

$$\sigma(P_i) = \sum_{e=1}^{E}\sum_{n=1}^{N} \Delta\mathbf{R}_{ni}^{e} \cdot \mathbf{t}_n^e - \sum_{e=1}^{E}\sum_{n=1}^{N} \Delta\mathbf{S}_{ni}^{e} \cdot \mathbf{u}_n^e \tag{12}$$

Eq. 12 is described in discretized formulation, where $\sigma(P_i)$ is the stress at Point P_i, $\Delta\mathbf{R}_{ni}^e$ and $\Delta\mathbf{S}_{ni}^e$ are the integrated kernel shape function products for node n at element e, $\mathbf{t}_n^e$ and $\mathbf{u}_n^e$ are the traction and displacement values at the boundary, respectively.
The procedure is explained for the same example of the previous chapter and again for LC4 (shown in Fig. 6). The stress is calculated at the same internal points for load cases 1 to 3. For load case 1 and 2 all points are internal points as shown in Fig. 6. There is no difficulty in the evaluation of the stress. For load case 3 some of the points of the excavated volume are boundary points. Because of the sharp corners at point A and B (shown in Fig. 9) the stress is infinite and a calculation directly at the point is not possible. To overcome this problem the stress is evaluated inside the adjacent element very near to the boundary, at an intrinsic coordinate of value $\xi = -0,90$. The stress is extrapolated to the boundary according to the parabolic shape function of the element. This is shown in Fig. 9.
The traction vector $\mathbf{t}$ is calculated using Eq. 13 in which the stress tensor σ is multiplied by the outward normal vector to the excavation surface $\mathbf{n}$.

$$\mathbf{t} = \sigma \cdot \mathbf{n} \tag{13}$$

The resulting traction at the excavation surface for LC4 is the sum of tractions obtained by internal stress evaluation for LC1 to LC3 completed by the tractions due to the virgin stress field. Once the loading tractions are found the solution for the current load step is evaluated by a single region boundary element calculation using Eq. 3.

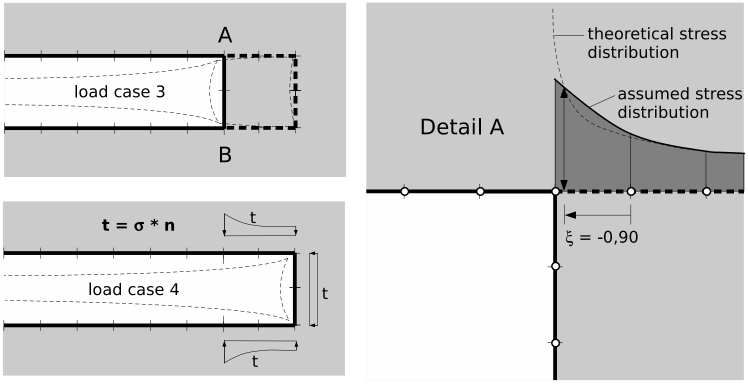

Figure 9: Treatment of singular point

3.4 Results - internal stress approach

The results of vertical displacements along the crown of the tunnel are shown in Fig. 10 for each load case (LC?_NEW). These results are compared with the reference solution (LC?_REF). As can be seen in Fig. 10 it seems that some loading is lost from load case to load case. The reason for this is the inaccurate evaluation of the tractions near to the singular points A and B shown in Fig. 9.

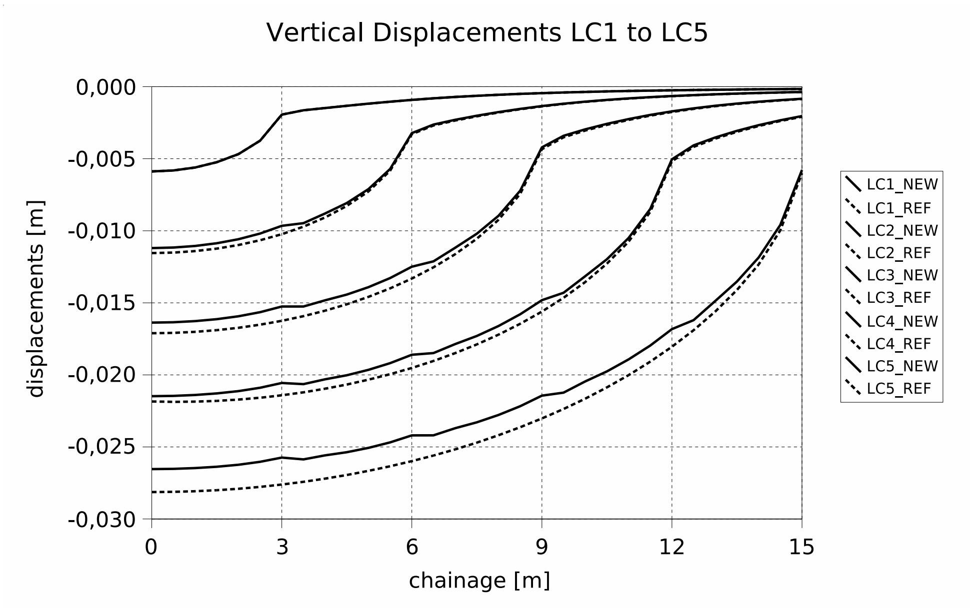

Figure 10: Vertical displacements at tunnel crown

4 CONCLUSIONS

Two new methods for the sequential excavation are shown. The main advantage of both methods against the conventional method of domain decomposition (MRBEM) is that a single boundary element region is necessary only to solve the problem. The simulation of excavation starts with a very small BEM mesh and this will extend from excavation step to step. Theoretically the number of steps is unlimited and has not to be defined prior to the calculation. The algorithm of internal stress evaluation is prefered to the one of internal displacements. With this method an intermediate calculation is not necessary and the excavation tractions can be calculated directly from the stress. Additional effort has to be spent for the correct evaluation of the stress distribution near to the boundary, where singular values of stress appear. Currently the implementation for 3D into the computercode BEFE++ is an ongoing task. A reasonable comparison of the efficiency to the conventional method of MRBEM only makes sense for a 3D example.

5 ACKNOWLEDGMENT

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