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A TIME DOMAIN BOUNDARY ELEMENT FORMULATION FOR A SIMPLIFIED POROELASTODYNAMIC CONTINUUM

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ABSTRACT

The dynamic responses of fluid-saturated porous continua subject to transient excitations such as seismic waves or ground vibrations are modeled with Biot's theory. The most natural choice of unknowns are the solid displacement and the pore pressure. However, in a time domain representation only a formulation with solid and fluid displacements is possible. But, under the assumption of a negligible inertia of the interstitial fluid a solid displacement/pore pressure formulation can be obtained. This approximation can be applied e.g. for soil.

Here, for this simplified dynamic Biot theory a boundary element formulation is given. First, the respective fundamental solutions are derived in Laplace domain with the method of Hörmander. These are implemented in the time domain BE formulation based on the Convolution Quadrature Method. For this formulation no time dependent fundamental solutions are necessary. Finally, an example show the applicability of the proposed formulation.

INTRODUCTION

In all versions of a poroelastic theory, the question arise which set of unknowns is used to formulate the set of governing differential equations. In the most general case, the vector of the solid displacement, the vector of the seepage velocity, and the pore pressure are used to derive the governing equations. In total there are seven degrees of freedom (dof) in a three-dimensional (3-d) formulation and five dof in a two-dimensional (2-d) formulation.

Either from a physical point of view as well as from a numerical point of view a reduction of these dof is desirable. Usually a fluid is described by a scalar value like the pressure and a solid by a vector quantity like the displacement vector. This can also be done here resulting in a sufficient set of unknowns [1], i.e., the solid displacement vector is chosen to describe the solid skeleton and the pore pressure for the fluid. However, this requires the elimination of the seepage velocity. Because the seepage velocity is given in a differential equation with respect to time by the balance law of the fluid, i.e., by Darcy's law, their elimination is only possible in a transformed domain, e.g., Laplace or Fourier domain [2, 3]. For modeling consolidation a quasi-static model is used, i.e., inertia effects are neglected, and, therefore, this elimination is even possible in time domain. However, the aimed application is wave propagation so such a simplification is not possible.

To avoid these difficulties in the Finite Element (FEM) literature on poroelastic wave propagation a simplified poroelastic model is introduced to be able to formulate and solve the governing differential equations directly in the time domain [4]. This simplification neglects only the inertia effects of the fluid but not those of the solid skeleton. In the following, this approach will be called *simple poro*. The applicability of this approach has been studied by Zienkiewicz et al. [5] showing that problems with low frequency accelerations can be treated well by this approach, e.g., applications in earthquake engineering.

In contrast to the FEM, for the Boundary Element Method

(BEM) no fundamental solution and, therefore, no BE formulation has been published for the simple poro model. This is due to the availability of a time domain formulation of the general poroelastic model [6]. This BE formulation is based on the Laplace domain fundamental solutions using the Convolution Quadrature Method proposed by Lubich [7, 8]. The usage of the Laplace domain solutions avoids any difficulties with the elimination of the seepage velocity. However, for treating also wave propagation problems in a non-linear poroelastic model, e.g., to take liquefaction into account, a coupled BE-FE procedure seems to be the best choice. But, for such a coupled formulation also a BE formulation for simple poro must be available.

In the next section, Biot's theory is recalled and the simplification is presented. For these governing equations fundamental solutions are derived using the method of Hörmander [9]. The next step, to establish a BE formulation is straight forward following exactly the procedure given for the general Biot equations [6, 10].

Throughout this paper, the summation convention is applied over repeated indices and Latin indices receive the values 1,2, and 1,2,3 in two-dimensions (2-d) and three-dimensions (3-d), respectively. Commas $()_{,i}$ denote spatial derivatives and, as usual, the Kronecker delta is denoted by δ_{ij}.

BIOT'S THEORY – GOVERNING EQUATIONS

Following Biot's approach to model the behavior of porous media, one possible representation of poroelastic constitutive equation is obtained using the total stress $\sigma_{ij} = \sigma^s_{ij} + \sigma^f \delta_{ij}$ and the pore pressure p as independent variables [11]. Introducing Biot's effective stress coefficient α and the solid displacement u_i the constitutive equation reads

$$\sigma_{ij} = G u_{i,j} + \left(K - \frac{2}{3}G\right) u_{k,k}\delta_{ij} - \alpha\delta_{ij} p \tag{1}$$

with the shear modulus and the compression modulus of the solid frame G and K, respectively. In this equation, a linear strain displacement relation is used, i.e., small deformation gradients are assumed. Additional to the total stress σ_{ij}, as a second constitutive equation the variation of fluid volume per unit reference volume ζ is introduced

$$\zeta = \alpha u_{k,k} + \frac{\phi^2}{R} p \tag{2}$$

with material constant R and the porosity ϕ. This variation of fluid ζ is defined by the mass balance over a reference volume, i.e., by the continuity equation

$$\frac{\partial \zeta}{\partial t} + q_{i,i} = a \tag{3}$$

with the specific flux $q_i = \phi w_i$, the seepage velocity w_i, and a source term $a(t)$.

Further, the balance of momentum for the bulk material must be fulfilled. Here, in the simplified theory the inertia of the fluid is neglected yielding

$$\sigma_{ij,j} + F_i = \rho \frac{\partial^2 u_i}{\partial t^2} , \tag{4}$$

with the bulk body force per unit volume F_i and the bulk density $\rho = \rho_s(1-\phi) + \phi\rho_f$ (ρ_s and ρ_f denotes the solid and fluid density, respectively).

Next, the fluid transport in the interstitial space expressed by the specific flux $q_i = \phi w_i$ is modeled with a simplified dynamic version of Darcy's law

$$\phi w_i = q_i = -\kappa \left(p_{,i} + \rho_f \frac{\partial^2 u_i}{\partial t^2} \right) , \tag{5}$$

where κ denotes the permeability.

To eliminate in the equations (1)-(5) the seepage velocity w_i, Darcy's law has to be rearranged to find an expression for the seepage velocity. With the above introduced simplification (neglection of the fluid inertia) it is possible to express with equation (5) the seepage velocity which yields subsequently the governing equations with this reduced set of unknowns

$$G u_{i,jj} + \left(K + \frac{1}{3}G\right) u_{j,ij} - \alpha p_{,i} - \rho \frac{\partial^2 u_i}{\partial t^2} = -F_i \tag{6a}$$

$$\kappa p_{,ii} - \frac{\phi^2}{R}\frac{\partial p}{\partial t} - \alpha \frac{\partial u_{i,i}}{\partial t} - \kappa\rho_f \frac{\partial^2 u_{i,i}}{\partial t^2} = -a \, . \tag{6b}$$

In [5], the authors discussed with the help of an analytical 1-d example the limitations of this simplification. Summarizing their results, in soil mechanics or geomechanical applications with mostly low frequency acceleration the complete Biot theory does not differ from the simplified form.

In the next section, fundamental solutions for the simplified Biot's equations are derived. These solutions will be later used in a Convolution Quadrature based BE formulation. Therefore, it is sufficient and to the authors knowledge the only possible way to deduce the fundamental solutions in Laplace domain. To do so, first, the set of governing equations (6) is transformed to Laplace domain, denoted by $\mathscr{L}\{f(t)\} = \hat{f}(s)$ with the complex Laplace variable s. Further, vanishing initial conditions are assumed. This leads in operator notation to $\mathbf{B}[\hat{u}_i \; \hat{p}]^T = [\hat{F}_i \; \hat{a}]^T$ with the not self-adjoint operator

$$\mathbf{B} = \begin{bmatrix} \left(G\nabla^2 - s^2\rho\right)\delta_{ij} + \left(K + \frac{1}{3}G\right)\partial_i\partial_j & -\alpha\partial_i \\ -s\left(\alpha - s\kappa\rho_f\right)\partial_i & \kappa\partial_{ii} - \frac{\phi^2 s}{R} \end{bmatrix} . \tag{7}$$

FUNDAMENTAL SOLUTIONS

A fundamental solution is mathematically spoken a solution of the equation $\mathbf{BG}+\mathbf{I}\delta(\mathbf{x}-\mathbf{y})=\mathbf{0}$ where the matrix of fundamental solutions is denoted by $\mathbf{G}$, the identity matrix by $\mathbf{I}$, and the Dirac distribution by $\delta(\mathbf{x}-\mathbf{y})$. Physically interpreted the solution at point $\mathbf{x}$ due to a single force at point $\mathbf{y}$ is looked for.

The operator type in (7) is an elliptical operator so the same method as for Biot's theory to find the fundamental solutions, the method of Hörmander [9], can be used. The idea of this method is to reduce the operator given in (7) to well known operators. Following this idea the definition of the inverse matrix operator $\mathbf{B}^{-1}=\mathbf{B}^{co}/\det(\mathbf{B})$ with the matrix of cofactors $\mathbf{B}^{co}$ is used. The ansatz $\mathbf{G}=\mathbf{B}^{co}\varphi$ for the matrix of fundamental solutions with an unknown scalar function φ inserted in the operator equation $\mathbf{BG}+\mathbf{I}\delta(\mathbf{x}-\mathbf{y})=\mathbf{0}$ yields to a more convenient representation of equations (7)

$$\begin{aligned}\mathbf{BB}^{co}\varphi+\mathbf{I}\delta(\mathbf{x}-\mathbf{y}) &= \det(\mathbf{B})\,\mathbf{I}\varphi+\mathbf{I}\delta(\mathbf{x}-\mathbf{y})=\mathbf{0}\\ &\rightsquigarrow\ \det(\mathbf{B})\,\varphi+\delta(\mathbf{x}-\mathbf{y})=0\,.\end{aligned}\tag{8}$$

With this reformulation, the search for a fundamental solution is reduced to solve the simpler scalar equation (8).

From the mathematical theory of Green's formula it is known that the fundamental solutions should satisfy the adjoint operator [12]. Opposite to elasticity the governing operator in poroelasticity is not self-adjoint. Therefore, here the solution for the adjoint operator $\mathbf{B}^\star$ is required.

Following formula (8), first, the determinant of the operator $\mathbf{B}^\star$ is calculated. This yields to the result

$$\det\mathbf{B}^\star=\kappa G^2\left(K+\frac{4}{3}G\right)\left(\nabla^2-\lambda_3^2\right)^2\left(\nabla^2-\lambda_1^2\right)\left(\nabla^2-\lambda_2^2\right)\tag{9}$$

with the roots $\lambda_i,\ i=1,2,3$

$$\begin{aligned}\lambda_{1,2}^2=\frac{1}{2}\Bigg[&\frac{\phi^2 s}{\kappa R}+\frac{\alpha s(\alpha-s\rho_f\kappa)}{\left(K+\frac{4}{3}G\right)\kappa}+\frac{s^2\rho}{K+\frac{4}{3}G}\\ &\pm\sqrt{\left(\frac{\phi^2 s}{\kappa R}+\frac{\alpha s(\alpha-s\rho_f\kappa)}{\left(K+\frac{4}{3}G\right)\kappa}+\frac{s^2\rho}{\left(K+\frac{4}{3}G\right)}\right)^2-4\frac{s^2\rho\phi^2 s}{R\left(K+\frac{4}{3}G\right)\kappa}}\ \Bigg]\\ \lambda_3^2=\frac{\rho s^2}{G}\,.&\end{aligned}\tag{10}$$

Expressing the determinant using this roots the scalar equation corresponding to (10) is given by

$$\left(\nabla^2-\lambda_3^2\right)\left(\nabla^2-\lambda_1^2\right)\left(\nabla^2-\lambda_2^2\right)\psi+\delta(\mathbf{x}-\mathbf{y})=0\tag{11}$$

using an appropriate abbreviation $\psi=G^2\kappa\left(K+\frac{4}{3}G\right)\varphi$. The solution of the modified higher order Helmholtz equation (11) is

$$\begin{aligned}\psi=\frac{1}{4\pi r}\Bigg[&\frac{e^{-\lambda_1 r}}{\left(\lambda_1^2-\lambda_2^2\right)\left(\lambda_1^2-\lambda_3^2\right)}+\frac{e^{-\lambda_2 r}}{\left(\lambda_2^2-\lambda_1^2\right)\left(\lambda_2^2-\lambda_3^2\right)}\\ &+\frac{e^{-\lambda_3 r}}{\left(\lambda_3^2-\lambda_2^2\right)\left(\lambda_3^2-\lambda_1^2\right)}\Bigg]\,.\end{aligned}\tag{12}$$

The distance between the two points $\mathbf{x}$ and $\mathbf{y}$ is denoted by $r=|\mathbf{x}-\mathbf{y}|$.

Having in mind that the Laplace transformation of the function describing a traveling wave front with constant speed c is $e^{-rs/c}=\mathscr{L}\{H(t-r/c)\}$ (in 3-d), it is obvious that the above solution (12) represents three waves. However, as the roots λ_i are functions of s, here, the wave speeds are time dependent representing the attenuation in a poroelastic continuum. This is in accordance with the well known three wave types of a poroelastic continuum [2]. The roots λ_1,λ_2, and λ_3 correspond to the wave velocities of the slow and fast compressional wave and to the shear wave, respectively.

The next steps are to insert the solution ψ back in the definition $\mathbf{G}=\mathbf{B}^{co}\varphi$ taking into account the relation between φ and ψ. After calculating the respective matrix of cofactors $\mathbf{B}^{co}$ the fundamental solutions are found

$$\begin{aligned}\mathbf{G}&=\begin{bmatrix}\hat{U}_{ij}^s & \hat{U}_i^f\\ \hat{P}_j^s & \hat{P}^f\end{bmatrix}\\ &=\frac{1}{G\kappa\left(K+\frac{4}{3}G\right)}\begin{bmatrix}\left(F\nabla^2+AD\right)\delta_{ij}-F\partial_{ij} & -A\alpha\partial_i\\ -AE\partial_i & A\left(B\nabla^2+A\right)\end{bmatrix}\psi\end{aligned}\tag{13}$$

with the abbreviations $A=G\nabla^2-s^2\rho,\ B=\left(K+\frac{1}{3}G\right),\ D=\kappa\nabla^2-\phi^2 s/R,\ E=s(\alpha-\rho_f\kappa),\ F=BD-C^2 s$. The explicit expressions for the fundamental solutions can be found in [13].

Comparing the fundamental solutions of Biot's complete theory the same solution is found with different λ_i but the singular behavior is equal.

BOUNDARY ELEMENT FORMULATION

The boundary integral equation based on Biot's theory can be written in the following form (for details see [6])

$$\begin{aligned}&\int_\Gamma\begin{bmatrix}U_{ij}^S & -P_j^S\\ U_i^F & -P^F\end{bmatrix}*\begin{bmatrix}t_i\\ q\end{bmatrix}\mathrm{d}\Gamma=\\ &\qquad\oint_\Gamma\begin{bmatrix}T_{ij}^S & Q_j^S\\ T_i^F & Q^F\end{bmatrix}*\begin{bmatrix}u_i\\ p\end{bmatrix}\mathrm{d}\Gamma+\begin{bmatrix}c_{ij} & 0\\ 0 & c\end{bmatrix}\begin{bmatrix}u_i\\ p\end{bmatrix}\end{aligned}\tag{14}$$

with the convolution integral $f * g = \int_0^t f(t-\tau)\, g(\tau)\, \mathrm{d}\tau$. In eq. (14), the total stress vector is denoted by t_i and the normal flux by q, whereas capital letters denote the respective fundamental solutions. Further, the domain is Ω with boundary Γ.

The integral free terms c_{ij} and c in (14) result from the limiting behavior of the fundamental solutions if $\mathbf{y}$ approaches $\mathbf{x}$. A series expansion of the fundamental solutions with respect to $r = |\mathbf{y}-\mathbf{x}|$ shows that $\hat{T}^S_{ij}$ and $\hat{Q}^F$ are strongly singular and in the limit $r \to 0$ equal to their elastostatic and acoustic counterparts, respectively. Therefore, the integral free terms c_{ij} and c are calculated as known from elastostatic and acoustic BE formulations and the first integral on the right hand side of (14) has to be defined in the sense of a Cauchy Principal Value (denoted by $\oint$). The other fundamental solutions are either regular, $\hat{P}^S_j$ and $\hat{U}^F_i$, or weakly singular $\hat{U}^S_{ij}, \hat{P}^F, \hat{T}^F_i$, and $\hat{Q}^S_j$. Clearly, also the time dependent counterparts of the fundamental solutions have the same singular behavior with respect to r (for details see [13]).

According to the boundary element method the boundary surface Γ is discretized by E elements Γ_e and for the state variables F polynomial shape functions $N^f_e(\mathbf{x})$ are defined. Further, the convolution integrals are approximated by the Convolution Quadrature Method [7,8]. Applying these approximations to the integral equation (14) results in the boundary element time stepping formulation for $n = 0, 1, \ldots, N$

$$\begin{bmatrix} c_{ij} u_i(n\Delta t) \\ c\, p(n\Delta t) \end{bmatrix} = \sum_{e,f=1}^{E,F} \sum_{k=0}^{n} \left\{ \begin{bmatrix} \omega^{ef}_{n-k}\Big(\hat{U}^S_{ij}\Big) & -\omega^{ef}_{n-k}\Big(\hat{P}^S_j\Big) \\ \omega^{ef}_{n-k}\Big(\hat{U}^F_i\Big) & -\omega^{ef}_{n-k}\Big(\hat{P}^F\Big) \end{bmatrix} \begin{bmatrix} t^{ef}_i(k\Delta t) \\ q^{ef}(k\Delta t) \end{bmatrix} - \begin{bmatrix} \omega^{ef}_{n-k}\Big(\hat{T}^S_{ij}\Big) & \omega^{ef}_{n-k}\Big(\hat{Q}^S_j\Big) \\ \omega^{ef}_{n-k}\Big(\hat{T}^F_i\Big) & \omega^{ef}_{n-k}\Big(\hat{Q}^F\Big) \end{bmatrix} \begin{bmatrix} u^{ef}_i(k\Delta t) \\ p^{ef}(k\Delta t) \end{bmatrix} \right\}. \tag{15}$$

The integration weights are calculated corresponding to

$$\omega^{ef}_n\Big(\hat{U}^S_{ij}\Big) = \frac{\mathscr{R}^{-n}}{L} \sum_{\ell=0}^{L-1} \int_\Gamma \hat{U}^S_{ij} \left(\frac{\gamma\Big(\mathscr{R} e^{i\ell\frac{2\pi}{L}}\Big)}{\Delta t} \right) \cdot {}^u N^f_e(\mathbf{x})\, \mathrm{d}\Gamma\, e^{-in\ell\frac{2\pi}{L}}, \tag{16}$$

respectively. A backward differential formula of order 2 (γ denotes the quotient of its characteristic polynomials) and the parameter choice $L = N$ and $\mathscr{R}^N = \sqrt{10^{-10}}$ yield the best results [14].

Note, the calculation of the integration weights is only based on the Laplace transformed fundamental solutions which are derived in the previous section. In order to arrive at a system of algebraic equations, point collocation is used.

According to $t - \tau = (n-k)\Delta t$, the integration weights ω^{ef}_{n-k} in (15) are only dependent on the difference $n-k$. This property is analogous to elastodynamic time domain BE formulations and can be used to establish a recursion formula for $n = 1, 2, \ldots, N$ $(m = n-k)$

$$\omega_0(\mathbf{C})\, \mathbf{d}^n = \omega_0(\mathbf{D})\, \bar{\mathbf{d}}^n + \sum_{m=1}^{n} \left(\omega_m(\mathbf{U})\, \mathbf{t}^{n-m} - \omega_m(\mathbf{T})\, \mathbf{u}^{n-m} \right) \tag{17}$$

with the time dependent integration weights ω_m containing the Laplace transformed fundamental solutions $\mathbf{U}$ and $\mathbf{T}$, respectively (see eq. (16)). Similarly, $\omega_0(\mathbf{C})$ and $\omega_0(\mathbf{D})$ are the corresponding integration weights of the first time step related to the unknown boundary data $\mathbf{d}^n$ and the known boundary data $\bar{\mathbf{d}}^n$ in time step n, respectively. Finally, a direct equation solver is applied where only the matrix of the first time step has to be inverted.

NUMERICAL EXAMPLES

To demonstrate that the results of the u-p formulation with neglect of the derivative of the seepage velocity are similar to the results of Biot's complete theory, the displacement response and the pore pressure distribution of a poroelastic half space in 3-d is compared. The material data are those of a soil ($E = 2.544 \cdot 10^8\, \frac{\mathrm{N}}{\mathrm{m}^2}, \nu = 0.298, \rho = 1884\, \frac{\mathrm{kg}}{\mathrm{m}^3}, \rho_f = 1000\, \frac{\mathrm{kg}}{\mathrm{m}^3}, \phi = 0.48, R = 1.2 \cdot 10^9\, \frac{\mathrm{N}}{\mathrm{m}^2}, \alpha = 0.98, \kappa = 3.55 \cdot 10^{-9}\, \frac{\mathrm{m}^4}{\mathrm{Ns}}$).

For the 3-d model of the half space a strip of $33\,\mathrm{m} \times 6\,\mathrm{m}$ has been discretized with 396 triangular linear elements on 238 nodes (see Fig. 1). The half space is loaded by a vertical total

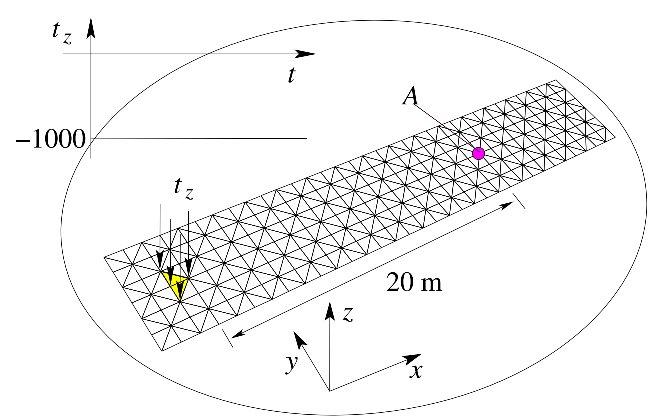

Figure 1. Poroelastic half space in 3-d: mesh and loading

stress vector $t_z = -1000\frac{N}{m^2} H(t)$ at an area of $1\,\mathrm{m}^2$ which is kept constant over the whole observation period. The remaining surface is traction free and assumed to be permeable, i.e., the pore pressure is zero all over the surface.

In figure 2, the calculated vertical displacement is plotted versus time at point A. Different to a 2-d calculation where no differences are visible [13] in 3-d some differences between the simplified theory and Biot's theory are visible. However, these differences are very small and in the range which can also be affected by numerics, i.e., also a change in the time step size can result in differences of the same order. So, in principle it can be concluded that in the 3-d calculation both formulations give the same result.

The pore pressure distribution in different depths (see the coordinates in Fig. 3 with origin at the tip of the load) is presented in Fig. 3. Due to the larger distance from the excitation point the fast compressional wave needs different times to reach the chosen points. The pore pressure does not vanish after the passage of the wave because the load is kept over the total observation period. Further, the pore pressure reduces with increasing depth as expected.

Finally, this comparison shows that the simplified theory can be used for the chosen material, a soil, and the presented excitations. There is no significant difference to Biot's complete theory. This confirms the results presented in [5].

CONCLUSIONS

Based on Biot's theory, a poroelastodynamic boundary element formulation with neglected derivative of the seepage velocity is presented for analyzing wave propagation in two and three-dimensional saturated porous continua. For a half space example, this formulation has been compared with a BE formulation based on Biot's complete theory. For the investigated material the solution from the complete u-p formulation and from the simplified poroelasticity are quite similar. Hence, for this example the influence of the derivative of the seepage velocity can be neglected.

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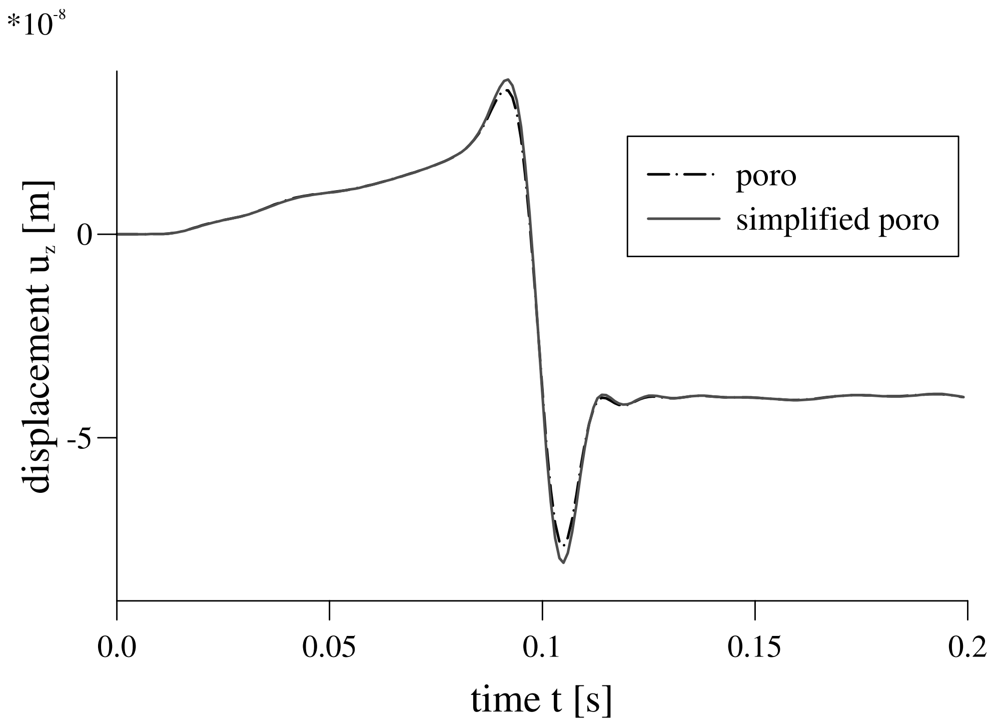

Figure 2. Vertical displacement at point A versus time

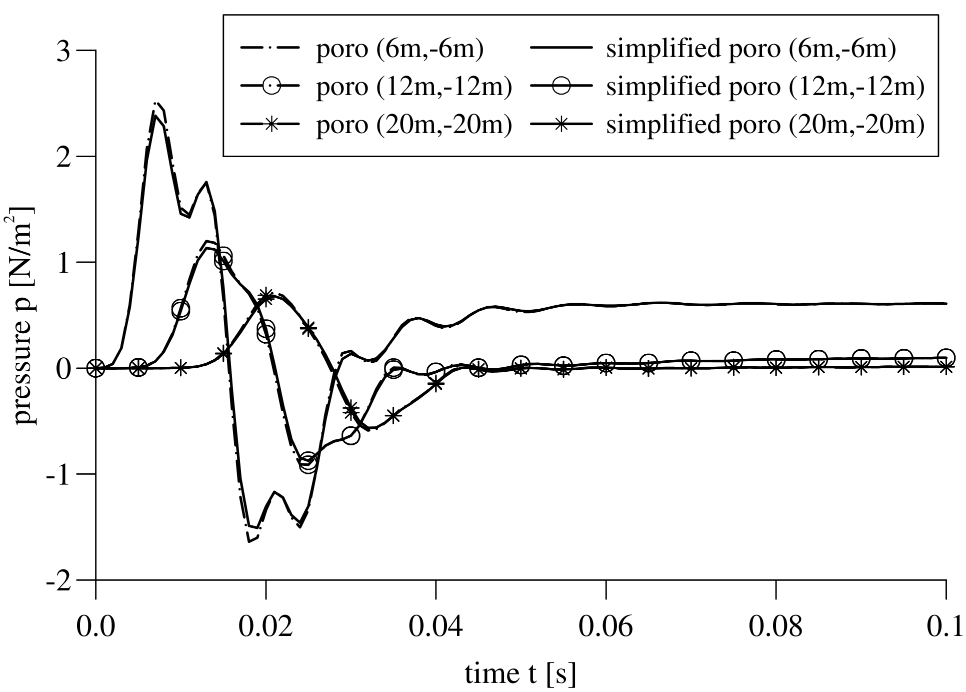

Figure 3. Pore pressure distribution below point A