Preparing Thin Layer Models for Application to the Finite Element Method (FEM)

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***Abstract*— Since the FEM is widely used and allows for reliable results in numerical modeling electromagnetic field problems, it will be used as a framework for implementation of analytically modeled layers of any material like metal sheets, conductive paint and the like. Based on analytical work done by [1] a model was developed to determine attenuation of electromagnetic waves due to reflection and absorption when traveling through thin layers of either dielectric or conductive and lossy media under different angles of incidence. The modeling is based on the transversal incident electric and magnetic field components of the incident plane wave and is adapted to the well known $\vec{A}, v$ - formulation for application to the Galerkin method ([3], [2]).**

I. Introduction

For the following analysis of the overall shielding effectiveness of thin layers the stack-up shall be free space followed by a thin layer and free space again.

The total shielding attenuation a_t is constituted by reflection attenuation a_r due to a reflection coefficient R on both surfaces of the thin layer and absorption attenuation a_a. Both a_r and a_a depend on all material parameters and the sum yields a_t as given in eqn. 7.

As far as the angles of incidence are concerned, two cases have to be distinguished. If the tangential magnetic field component of the incident wave is parallel to the surface (TM-case), as shown in figure I, the effective tangential components are given by

$$\vec{E}_t = E_0(1+R)cos\varphi, \quad \vec{H}_t = \frac{\vec{E}_0}{Z_0}(1-R). \qquad (1)$$

For the TE-case the following equations apply

$$\vec{E}_t = E_0(1+R), \quad \vec{H}_t = \frac{\vec{E}_0}{Z_0}(1-R)cos\varphi. \qquad (2)$$

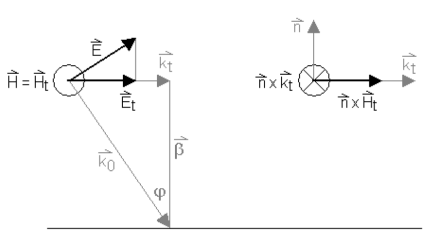

Fig. 1. Transversal magnetic (TM) wave model.

In figure I $\vec{H}_t$ and $\vec{E}_t$ denote the tangential components of the incident plane wave, $\vec{k_0}, \vec{k_t}$ and $\vec{\beta}$ are the wave vector, its tangential vector and its normal vector and the angle of incidence φ.

For the reflection coefficient the impedances of the thin layer and free space have to be determined. A common formula applicable to dielectric and lossy media as well reads

$$Z_m = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r - j\frac{\sigma}{\omega}}} \qquad (3)$$

and has been introduced by [4]. With eqn. 1 and eqn. 2 the reflection coefficient R reads

$$R = \frac{Z_m\frac{\beta}{k_0} - Z_0cos(\varphi)}{Z_m\frac{\beta}{k_0} + Z_0cos(\varphi)} \qquad (4)$$

with Z_0 being the impedance of free space. The wave number k_0 is given by

$$k_0 = |\vec{k_0}| = \omega\sqrt{\epsilon_0\mu_0} \qquad (5)$$

The known matrix formulation for the transmission of electromagnetic waves through a thin layer (see also [3]) is

$$\begin{pmatrix} \vec{E_{t+}} \\ \vec{n} \times \vec{H_{t+}} \end{pmatrix} = \begin{pmatrix} \bar{a_{11}} & \bar{a_{12}} \\ \bar{a_{21}} & \bar{a_{22}} \end{pmatrix} \cdot \begin{pmatrix} \vec{E_{t-}} \\ \vec{n} \times \vec{H_{t-}} \end{pmatrix}. \qquad (6)$$

The tangential incident field components $\vec{E_{t-}}$ and $\vec{H_{t-}}$ must be replaced by the respective effective tangential components for the TM- or TE-case. The tangential field components $\vec{E_{t+}}$ and $\vec{H_{t+}}$ represent the wave components exiting the thin layer. The total attenuation is finally given by

$$a_t = a_r + a_a + a_r. \qquad (7)$$

II. Conclusions

A model to determine the attenuation of plane waves traveling through thin layers has been presented. Since angles of incidence of electromagnetic waves are, in general, beyond control, it is indispensable to find and define limits of the applicability of such a model by means of the FEM in order to use it in real world shielding applications.

References

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