Resonant magnetic field perturbations and the quasilinear response of the tokamak plasma*

M. F. Heyn[1], I. B. Ivanov[1,2], S. V. Kasilov[1,3], W. Kernbichler[1], P. Leitner[1]

[1]*Association EURATOM-ÖAW, Institut für Theoretische Physik - Computational Physics, TU Graz, Austria*

[2]*Petersburg Nuclear Physics Institute, Russia*

[3]*Institute of Plasma Physics, National Science Center "Kharkov Institute of Physics and Technology", Ukraine*

Introduction

The torque from resonant magnetic field perturbations (RMPs) acting on the plasma tends to modify the plasma rotation such that the frequency of the perturbation in the rest frame of the electron fluid is minimized. If the RMP amplitude is large enough, a "locked mode" is formed – a stable state where this frequency is close to zero and shielding of RMP by the plasma is reduced. In this case, even for relatively small RMP amplitudes, large islands can be produced such that linear and quasilinear (QL) theories are not valid any more. However, the violation of QL theory does not necessarily prohibit its use for the description of the locked mode. Due to a mass difference, nonlinear effects become significant for electron component at much smaller perturbation field amplitudes than for ions. Therefore, it is possible that QL theory is still valid for ions while electrons are in the nonlinear regime. However, the consequence of this regime is that electrons are excluded from RMP shielding and are not important anymore - the same result as for QL theory and this allows us to use QL theory for both components.

In the present report, the formation of a locked mode in the case of small RMP amplitudes is considered assuming linear theory still to be valid. The effect of RMP on rotation velocity, density and temperature profiles is described by a set of quasi-linear transport equations. The kinetic approach [1,2] is used where Maxwell equations are solved for a straight cylindrical tokamak model using the linear plasma response currents computed in the kinetic approximation simplified by a specific finite Larmor radius expansion. Here, the collisionless case is considered for the plasma response to RMP which gives also a reasonable estimate of the perturbation fields in presence of collisions [2].

*This work, supported by the European Communities under the contract of Association between EURATOM and the Austrian Academy of Sciences, was carried out within the framework of the European Fusion Development Agreement. The views and opinions expressed herein do not necessarily reflect those of the European Commission. Additional funding is provided by the Austrian Science Foundation, FWF, under contract number P19889-N16.

Basic equations

The effect of the perturbation field on the background parameters in the absence of other transport effects is described by the quasilinear kinetic equation. In canonical action-angle variables θ^α and J_α this equation becomes

$$\frac{\partial f_0}{\partial t} = \frac{\pi}{2}\sum_m m\cdot\frac{\partial}{\partial J}\left|H_m\right|^2\delta(m\cdot\Omega-\omega)\,m\cdot\frac{\partial f_0}{\partial J}. \tag{1}$$

Here J, Ω and m denote the whole set of canonical actions, canonical frequencies and wavenumbers of Fourier expansion of the perturbed Hamiltonian over canonical angles, H_m are the coefficients of this expansion and ω is the perturbation frequency. Taking moments of this equation and adding the contributions of other transport effects, balance equations are obtained which describe the temporal evolution of the plasma density n_e, the toroidal ion rotation velocity V_i^φ, and the electron and ion temperatures $T_\mathrm{e,i}$,

$$\frac{\partial n_\mathrm{e}}{\partial t} + \frac{1}{S}\frac{\partial}{\partial r}S\left(\Gamma_\mathrm{e}^{(\mathrm{EM})}+\Gamma_\mathrm{e}^{(\mathrm{A})}\right) = S_{n_\mathrm{e}}, \tag{2}$$

$$\begin{aligned}
&m_\mathrm{i}n_\mathrm{i}\left\langle g_{\varphi\varphi}\right\rangle\frac{\partial V_\mathrm{i}^\varphi}{\partial t} - \frac{1}{S}\frac{\partial}{\partial r}S m_\mathrm{i}n_\mathrm{i}\left\langle g_{\varphi\varphi}\right\rangle\mu_\varphi^{(\mathrm{A})}\frac{\partial V_\mathrm{i}^\varphi}{\partial r}\\
&\qquad+\frac{1}{c}\sqrt{g}B_0^\vartheta\sum_{\mathrm{e,i}}e_\mathrm{e,i}\left(\Gamma_\mathrm{e,i}^{(\mathrm{EM})}+\Gamma_\mathrm{e,i}^{(\mathrm{A})}\right) = S_{p_\varphi}^{(\mathrm{NBI})},
\end{aligned}\tag{3}$$

$$\begin{aligned}
\frac{\partial T_\mathrm{e,i}}{\partial t} &= -\frac{T_\mathrm{e,i}}{n_\mathrm{e,i}}\left[S_{n_\mathrm{e,i}} - \frac{1}{S}\frac{\partial}{\partial r}S\left(\Gamma_\mathrm{e,i}^{(\mathrm{EM})}+\Gamma_\mathrm{e,i}^{(\mathrm{A})}\right)\right]\\
&\quad+\frac{2}{3}\frac{1}{n_\mathrm{e,i}}\left[e_\mathrm{e,i}\frac{\partial\Phi}{\partial r}\Gamma_\mathrm{e,i}^\mathrm{EM}+S_{nT;\mathrm{e,i}}^{(\mathrm{A})}+S_{nT;\mathrm{e,i}}^{(\mathrm{AUX})}-\frac{1}{S}\frac{\partial}{\partial r}S\left(Q_\mathrm{e,i}^{(\mathrm{EM})}+Q_\mathrm{e,i}^{(\mathrm{A})}+Q_\mathrm{e,i}^{(\mathrm{NEO})}\right)\right].
\end{aligned}\tag{4}$$

Here, r is the effective radius, S is the flux surface area, $\langle\ldots\rangle$ denotes neoclassical flux surface average, $g_{\varphi\varphi}$ and g are the covariant toroidal component of the metric tensor and its determinant respectively, and B_0^ϑ, Φ, m_i, $\mu_\varphi^{(\mathrm{A})}$, $\Gamma_\mathrm{e,i}^{(\mathrm{A})}$ and $Q_\mathrm{e,i}^{(\mathrm{A})}$ and $S_b^{(a)}$ are the contra-variant poloidal component of the main magnetic field, equilibrium electrostatic potential, ion mass, anomalous toroidal viscosity coefficient, anomalous particle and heat flux densities and various sources, respectively. The neoclassical heat flux density $Q_\mathrm{i}^{(\mathrm{NEO})}$ is considered for ions only. The quasilinear particle and heat flux densities are composed of thermodynamic forces $\mathscr{A}_i$ and diffusion coefficients D_{ij} in the following way,

$$\Gamma_\mathrm{e,i}^\mathrm{EM} = -n_\mathrm{e,i}(D_{11}\mathscr{A}_1+D_{12}\mathscr{A}_2) \tag{5}$$

$$Q_\mathrm{e,i}^\mathrm{EM} = -n_\mathrm{e,i}T_\mathrm{e,i}(D_{21}\mathscr{A}_1+D_{22}\mathscr{A}_2), \tag{6}$$

where

$$\mathscr{A}_1 = \frac{1}{n_\mathrm{e,i}}\frac{\partial n_\mathrm{e,i}}{\partial r}+\frac{e}{T_\mathrm{e,i}}\frac{\partial\Phi}{\partial r}-\frac{3}{2T_\mathrm{e,i}}\frac{\partial T_\mathrm{e,i}}{\partial r}, \qquad \mathscr{A}_2 = \frac{1}{T_\mathrm{e,i}}\frac{\partial T_\mathrm{e,i}}{\partial r}, \tag{7}$$

and the diffusion coefficients for a single perturbation mode D_{ij} are given in the lowest Larmor radius expansion order by

$$D_{11} = \sqrt{\pi}\left(\frac{v_T}{B_0}\right)^2 \left|\frac{Z}{\omega_E}\right|^3 \mathrm{e}^{-Z^2} \left|\omega_E \tilde{B}^r + ck_\perp \tilde{E}_\parallel\right|^2, \tag{8}$$

$$D_{12} = D_{21} = (1+Z^2)D_{11}, \qquad D_{22} = \left[1 + \left(1+Z^2\right)^2\right] D_{11}.$$

Here $Z = \omega_E(\sqrt{2}k_\parallel v_T)^{-1}$, ω_E is the single RMP mode frequency in the moving frame where the radial electric field vanishes, $\tilde{B}^r$ and $\tilde{E}_\parallel$ are radial component of the perturbation magnetic field and parallel component of the perturbation electric field, respectively (see also [3]). Coefficients (8) should be summed up over the modes of the perturbation.

Quasilinear modelling

For the numerical solution, the balance equations are discretized in space on the radial grid and solved following the time evolution with an implicit time stepping method. Each time step, the perturbation electromagnetic field is updated by the kinetic wave code KILCA [2]. For the present modelling, tokamak parameters are taken for a JET-scale device with the RMP spectrum typical for experiments on ELM mitigation using the C-coil. The amplitudes of this field are taken corresponding to 20 kA and 5 kA coil current mentioned below as “high” and “medium”, respectively. Results are obtained for the whole spectrum as well as for a single perturbation mode $n = 1$, $m = -3$. This mode together with the $n = 1$, $m = -2$ mode are the main contributions to the spectrum. For both, the “high” and the “medium” amplitude model, locking is obtained, i.e. radial electric field is altered in such a way that the perpendicular electron fluid velocity becomes zero at the resonant surface. In addition, in both cases a quasilinear plateau in the temperature profile is formed in the resonant layer for the electrons and partly for the ions. However, the toroidal rotation velocity remains finite for the medium amplitude in contrast to the high amplitude case where a quasilinear plateau is formed also on the density. This completely stops the rotation at the resonant surface. Only in this case the perturbation field changes significantly in the whole volume tending to recover the vacuum value while in the medium amplitude case this change is localized to the resonant layer. This difference is related to the ion contribution of RMP shielding. The electron component which provides the main shielding effect is more sensitive to quasilinear plateau formation because its parallel current is highly localized at the resonant surface where the quasilinear effect is strongest. Thus, the parallel electron response current is almost completely eliminated by this effect. However, shielding by the ions still remains efficient for the medium amplitude case and is reduced strongly only in the high amplitude case. Note that for a proper description of the ion component contribution to RMP shielding the kinetic approach is important because the width of the resonant layer is usually comparable with the ion Larmor radius.

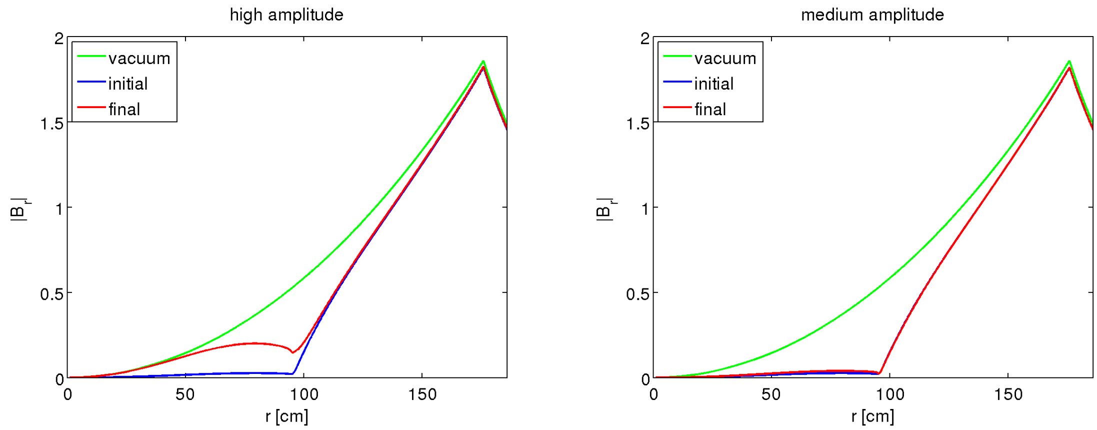

Fig. 1. Radial profiles of the Fourier amplitude of the radial magnetic field for high (left) and medium (right) amplitude perturbations without plasma response (green) and with plasma response for the unperturbed (blue) and the perturbed (red) equilibrium plasma.

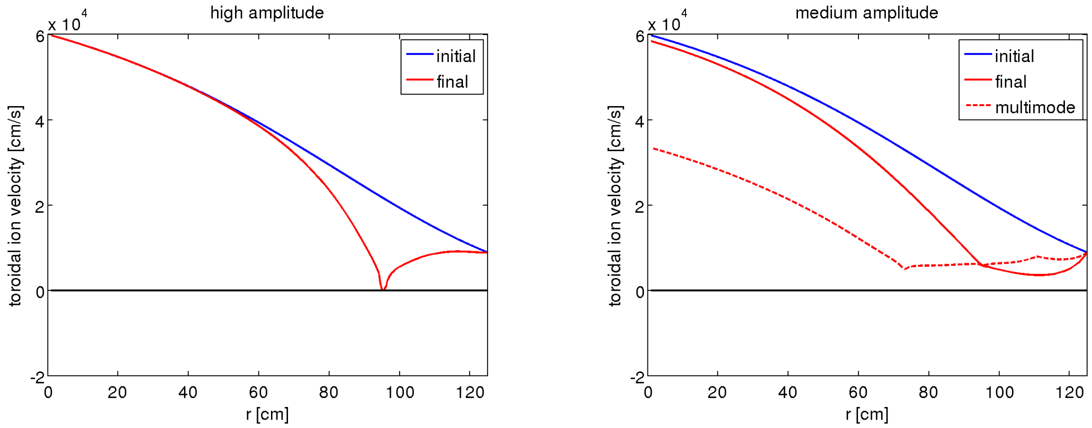

Fig. 2. Profiles of the toroidal rotation velocity for high (left) and medium (right) perturbation amplitudes for the unperturbed (blue) and the perturbed (red) equilibrium plasma. Solid and dashed lines correspond to a single mode and a full coil spectrum respectively.

References

[1] M. F. Heyn, I. B. Ivanov, S. V. Kasilov, et al, Nucl. Fusion **46**, S159 (2006).

[2] I. B. Ivanov, M. F. Heyn, S. V. Kasilov, and W. Kernbichler, Phys. Plasmas **18**, 022501-1 (2011).

[3] I. B. Ivanov, S. V. Kasilov, W. Kernbichler, et al, JETP Letters **86**, 364 (2007).