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Consistent Poroelastodynamic Plate-Theories

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Contents

Motivation

- Mathematical modelling of sound insulation by porous plates
- Mathematical modelling of the dynamical behaviour of poroelastic plates
- In classical theories, kinematic assumptions are introduced
  - $\longrightarrow$ Kirchhoff plate
  - $\longrightarrow$ Mindlin plate

Motivation

- Mathematical modelling of sound insulation by porous plates
- Mathematical modelling of the dynamical behaviour of poroelastic plates
- In classical theories, kinematic assumptions are introduced
  - $\longrightarrow$ Kirchhoff plate
  - $\longrightarrow$ Mindlin plate
- Can the classical assumptions be transferred to poroelasticity, especially to the pore pressure?
- An assumption–free derivation is used

Contents

The poroelastic continuum

- Biot's theory of poroelasticity in frequency domain
  $\rightarrow$ **u**, p as degrees of freedom
- porosity : $\phi = \frac{V^f}{V}$; full saturation assumed

$$\mu \Delta \hat{\mathbf{u}} + (\mu + \lambda) \nabla\nabla \cdot \hat{\mathbf{u}} - (\alpha - \beta) \nabla \hat{p} + \omega^2 (\rho - \rho_f) \hat{\mathbf{u}} = \beta \hat{\mathbf{f}}^f - \hat{\mathbf{F}}$$

$$\frac{\beta}{\mathrm{i}\omega\rho_f} \Delta \hat{p} - \mathrm{i}\omega \frac{\phi^2}{R} \hat{p} - \mathrm{i}\omega(\alpha - \beta) \nabla \cdot \hat{\mathbf{u}} = \frac{\beta}{\mathrm{i}\omega\rho_f} \hat{\mathbf{f}}^f$$

The poroelastic continuum

- Biot's theory of poroelasticity in frequency domain
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- porosity : $\phi = \frac{V^f}{V}$; full saturation assumed

$$\mu\Delta\hat{\mathbf{u}} + (\mu+\lambda)\nabla\nabla\cdot\hat{\mathbf{u}} - (\alpha-\beta)\nabla\hat{p} + \omega^2(\rho-\rho_f)\hat{\mathbf{u}} = \beta\hat{\mathbf{f}}^f - \hat{\mathbf{F}}$$

$$\frac{\beta}{\mathrm{i}\omega\rho_f}\Delta\hat{p} - \mathrm{i}\omega\frac{\phi^2}{R}\hat{p} - \mathrm{i}\omega(\alpha-\beta)\nabla\cdot\hat{\mathbf{u}} = \frac{\beta}{\mathrm{i}\omega\rho_f}\hat{\mathbf{f}}^f$$

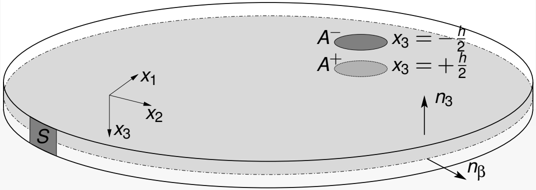
- Total energy stored in the system : $\Pi = \Pi(\mathbf{u}, p)$
- A variation from a state of equilibrium involves no change in energy

$$\delta\Pi = \int_\Omega \delta U_\Omega \, \mathrm{d}\Omega + \int_\Gamma \delta U_\Gamma \, \mathrm{d}\Gamma \stackrel{!}{=} 0$$

$$\delta\Pi = \delta\Pi(\mathbf{u}, \delta\mathbf{u}, p, \delta p) \qquad \delta U = \delta U(\mathbf{u}, \delta\mathbf{u}, p, \delta p)$$

- An integration over the thickness coordinate is needed to deduce the plate equations

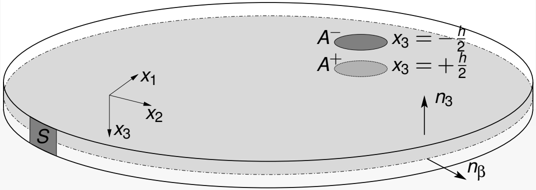
- Geometry of the plate



$$\mathbf{u} = \mathbf{u}(\mathbf{x}, \omega)$$

$$p = p(\mathbf{x}, \omega)$$

- Geometry of the plate



$$\mathbf{u} = \mathbf{u}(\mathbf{x}, \omega)$$
$$p = p(\mathbf{x}, \omega)$$

- Substitution of **u** and p by a power series in x_3-direction

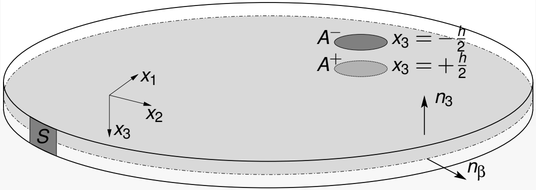
$$\mathbf{u}(x_1, x_2, x_3) = \sum_{k=0}^{\infty} \overset{k}{\mathbf{u}}(x_1, x_2)\, x_3^k$$

$$p(x_1, x_2, x_3) = \sum_{k=0}^{\infty} \overset{k}{p}(x_1, x_2)\, x_3^k$$

$\overset{k}{\mathbf{u}}(x_1, x_2), \overset{k}{p}(x_1, x_2) \ldots$ Unknown functions of order k

Deducing the plate-problem

- Geometry of the plate



$$\mathbf{u} = \mathbf{u}(\mathbf{x}, \omega)$$
$$p = p(\mathbf{x}, \omega)$$

- Substitution of $\delta\mathbf{u}$ and δp by a power series in x_3-direction

$$\delta\mathbf{u}(x_1, x_2, x_3) = \sum_{\ell=0}^{\infty} \delta\overset{\ell}{\mathbf{u}}(x_1, x_2)\, x_3^{\ell}$$

$$\delta p(x_1, x_2, x_3) = \sum_{\ell=0}^{\infty} \delta\overset{\ell}{p}(x_1, x_2)\, x_3^{\ell}$$

$$\delta\overset{\ell}{\mathbf{u}}(x_1, x_2), \delta\overset{\ell}{p}(x_1, x_2) \ldots \text{Unknown functions of order } \ell$$

- Integration over the plate thickness

$$\int_{\Omega} \delta U_{\Omega}\left(\mathbf{u}, \delta\mathbf{u}, p, \delta p\right)\,\mathrm{d}\Omega$$

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$$\sum_{k=0}^{\infty}\sum_{\ell=0}^{\infty}\int\limits_{A}\int\limits_{-\frac{h}{2}}^{\frac{h}{2}} \delta U_{\Omega}\left(\overset{k}{\mathbf{u}}, \delta\overset{\ell}{\mathbf{u}}, \overset{k}{p}, \delta\overset{\ell}{p}, \mathcal{P}(x_3)\right)\, \mathrm{d}x_3\, \mathrm{d}A$$

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$$\rightarrow \quad \sum_{k=0}^{\infty}\sum_{\ell=0}^{\infty}\int\limits_{A} \delta U_{\Omega}\left(\overset{k}{\mathbf{u}}, \delta\overset{\ell}{\mathbf{u}}, \overset{k}{p}, \delta\overset{\ell}{p}, \mathcal{P}(h)\right) \,\mathrm{d}A$$

- Integration over the plate thickness

$$\int\limits_{\Omega} \delta U_\Omega\left(\mathbf{u}, \delta\mathbf{u}, p, \delta p\right)\, \mathrm{d}\Omega$$

$$\sum_{k=0}^{\infty}\sum_{\ell=0}^{\infty}\int\limits_{A}\int\limits_{-\frac{h}{2}}^{\frac{h}{2}} \delta U_\Omega\left(\overset{k}{\mathbf{u}}, \delta\overset{\ell}{\mathbf{u}}, \overset{k}{p}, \delta\overset{\ell}{p}, \mathcal{P}(x_3)\right)\, \mathrm{d}x_3\, \mathrm{d}A$$

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- Extract plate problem (identify and decouple plate and disc quantities)

Approximation by power series

- Integration over the plate thickness

$$\int_{\Omega} \delta U_{\Omega}\left(\mathbf{u}, \delta\mathbf{u}, p, \delta p\right)\,\mathrm{d}\Omega$$

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- Extract plate problem (identify and decouple plate and disc quantities)
- Poroelastic case does not impinge on the decoupling of plate and disc problem
- Truncation of the power series

Approximation by power series

- The plate quantities
  - $\overset{k}{u}_3 \quad \longrightarrow \quad k := 0, 2, 4, \ldots$ Vertical displacement of the cross section ($\overset{k}{w}$)
  - $\overset{k}{u}_\alpha \quad \longrightarrow \quad k := 1, 3, 5, \ldots$ Rotation of the cross section ($\overset{k}{\psi}_\alpha$)
  - $\overset{k}{p} \quad \longrightarrow \quad k := 1, 3, 5, \ldots$ Pore pressure distribution over the cross section

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- Truncation with respect to a specific order of k

- Truncation with respect to the order of the plate thickness h plate parameter

$$\left(c^2\right)^n = \left(\frac{h^2}{12}\right)^n \quad n \in \mathbb{N}$$

  - $n = 0 \quad \longrightarrow$ Theory of zeroth order
  - $n = 1 \quad \longrightarrow$ Theory of first order
  - $n = 2 \quad \longrightarrow$ Theory of second order

The PDE operators

Zeroth order $\quad \mathcal{L}^{0}_{3\times3}\mathbf{u} = \mathbf{f} \quad$ with $\quad \mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}]^\top$

First order $\quad \mathcal{L}^{1}_{6\times6}\mathbf{u} = \mathbf{f} \quad$ with $\quad \mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w}, \overset{3}{\boldsymbol{\psi}}, \overset{3}{p}]^\top$

Second order $\quad \mathcal{L}^{2}_{9\times9}\mathbf{u} = \mathbf{f} \quad$ with $\quad \mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w}, \overset{3}{\boldsymbol{\psi}}, \overset{3}{p}, \overset{4}{w}, \overset{5}{\boldsymbol{\psi}}, \overset{5}{p}]^\top$

The PDE operators

Zeroth order $\mathcal{L}^{0}_{3\times 3}\mathbf{u} = \mathbf{f}$ with $\mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}]^{\top}$

$\longrightarrow$ rigid body motions

First order $\mathcal{L}^{1}_{6\times 6}\mathbf{u} = \mathbf{f}$ with $\mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w}, \overset{3}{\boldsymbol{\psi}}, \overset{3}{p}]^{\top}$

$\longrightarrow$ Fourth order PDE

Second order $\mathcal{L}^{2}_{9\times 9}\mathbf{u} = \mathbf{f}$ with $\mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w}, \overset{3}{\boldsymbol{\psi}}, \overset{3}{p}, \overset{4}{w}, \overset{5}{\boldsymbol{\psi}}, \overset{5}{p}]^{\top}$

$\longrightarrow$ Sixth order PDE

The PDE operators

Zeroth order $\mathcal{L}^0_{3\times3}\mathbf{u} = \mathbf{f}$ with $\mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}]^\top$

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First order $\mathcal{L}^1_{6\times6}\mathbf{u} = \mathbf{f}$ with $\mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w}, \overset{3}{\boldsymbol{\psi}}, \overset{3}{p}]^\top$

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$\longrightarrow$ Sixth order PDE

- The factor c^2 has to be involved when reducing the system, e.g.

$$c^2(\nabla \overset{0}{w} + \overset{1}{\boldsymbol{\psi}}) = \mathcal{O}(c^4) \approx 0$$

$$\nabla \overset{0}{w} + \overset{1}{\boldsymbol{\psi}} = \mathcal{O}(c^2)$$

The PDE operators

Zeroth order $\mathcal{L}^0_{3\times3}\mathbf{u} = \mathbf{f}$ with $\mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}]^\top$

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First order $\mathcal{L}^1_{6\times6}\mathbf{u} = \mathbf{f}$ with $\mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w}, \overset{3}{\boldsymbol{\psi}}, \overset{3}{p}]^\top$

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$$c^2(\nabla \overset{0}{w} + \overset{1}{\boldsymbol{\psi}}) = \mathcal{O}(c^4) \approx 0$$
$$\nabla \overset{0}{w} + \overset{1}{\boldsymbol{\psi}} = \mathcal{O}(c^2)$$

- Problems arise when trying to solve the full system right away
  $\longrightarrow$ Reducing before solving

The PDE operators

Linear ansatz in k $\mathcal{L}_{3\times3}\mathbf{u} = \mathbf{f}$ with $\mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}]^\top$

Quadratic ansatz in k $\mathcal{L}_{4\times4}\mathbf{u} = \mathbf{f}$ with $\mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w},]^\top$

Cubic ansatz in k $\mathcal{L}_{6\times6}\mathbf{u} = \mathbf{f}$ with $\mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w}, \overset{3}{\boldsymbol{\psi}}, \overset{3}{p}]^\top$

The PDE operators

Linear ansatz in k $\quad \mathcal{L}_{3\times3}\mathbf{u} = \mathbf{f} \quad \text{with} \quad \mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}]^\top$

$\longrightarrow$ sixth order PDE

Quadratic ansatz in k $\quad \mathcal{L}_{4\times4}\mathbf{u} = \mathbf{f} \quad \text{with} \quad \mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w},]^\top$

$\longrightarrow$ eighth order PDE

Cubic ansatz in k $\quad \mathcal{L}_{6\times6}\mathbf{u} = \mathbf{f} \quad \text{with} \quad \mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w}, \overset{3}{\boldsymbol{\psi}}, \overset{3}{p}]^\top$

$\longrightarrow$ twelfth order PDE

Linear ansatz in k $\quad \mathcal{L}_{3\times 3}\mathbf{u} = \mathbf{f} \quad \text{with} \quad \mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}]^\top$

$\longrightarrow$ sixth order PDE

Quadratic ansatz in k $\quad \mathcal{L}_{4\times 4}\mathbf{u} = \mathbf{f} \quad \text{with} \quad \mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w},]^\top$

$\longrightarrow$ eighth order PDE

Cubic ansatz in k $\quad \mathcal{L}_{6\times 6}\mathbf{u} = \mathbf{f} \quad \text{with} \quad \mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w}, \overset{3}{\boldsymbol{\psi}}, \overset{3}{p}]^\top$

$\longrightarrow$ twelfth order PDE

- System can be solved as a whole, without beeing reduced

The PDE operators

Linear ansatz in k $\quad \mathcal{L}_{3\times 3}\mathbf{u} = \mathbf{f} \quad \text{with} \quad \mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}]^\top$

$\longrightarrow$ sixth order PDE

Quadratic ansatz in k $\quad \mathcal{L}_{4\times 4}\mathbf{u} = \mathbf{f} \quad \text{with} \quad \mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w},]^\top$

$\longrightarrow$ eighth order PDE

Cubic ansatz in k $\quad \mathcal{L}_{6\times 6}\mathbf{u} = \mathbf{f} \quad \text{with} \quad \mathbf{u} = [\overset{0}{w}, \overset{1}{\boldsymbol{\psi}}, \overset{1}{p}, \overset{2}{w}, \overset{3}{\boldsymbol{\psi}}, \overset{3}{p}]^\top$

$\longrightarrow$ twelfth order PDE

- System can be solved as a whole, without beeing reduced
- At least a quadratic ansatz in k is needed to model a Kirchhoff-type equation (extended by higher order terms)

Contents

- The reduced *First Order Problem* with $\overset{0}{w} := w$ and $\overset{1}{p} := p$

$$\begin{bmatrix} D\Delta\Delta - h\omega^2\rho_\beta & h(B_1\Delta - \beta) \\ \mathrm{i}\omega h(B_1\Delta - \beta) & -hB_2 \end{bmatrix} \begin{bmatrix} w \\ p \end{bmatrix} = \begin{bmatrix} F \\ Q \end{bmatrix}$$

- The reduced *First Order Problem* with $\overset{0}{w} := w$ and $\overset{1}{p} := p$

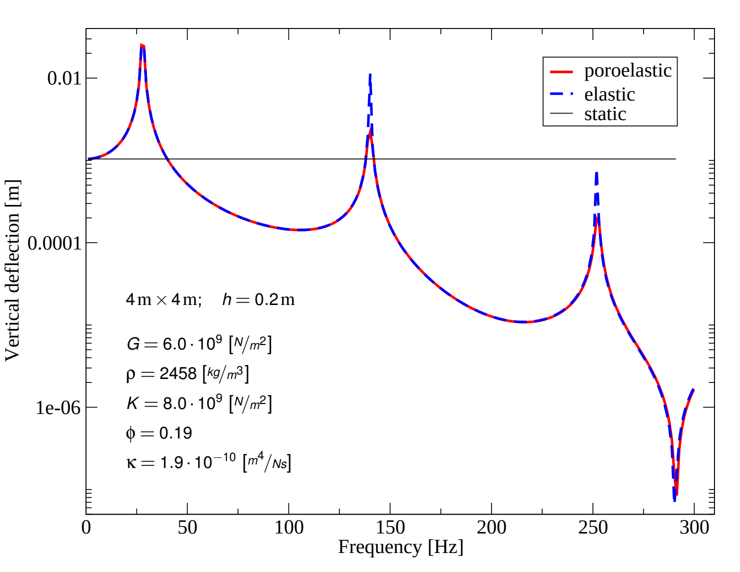
$$\begin{bmatrix} D\Delta\Delta - h\omega^2\rho_\beta & h(B_1\Delta - \beta) \\ i\omega h(B_1\Delta - \beta) & -hB_2 \end{bmatrix} \begin{bmatrix} w \\ p \end{bmatrix} = \begin{bmatrix} F \\ Q \end{bmatrix}$$

- Weak form :

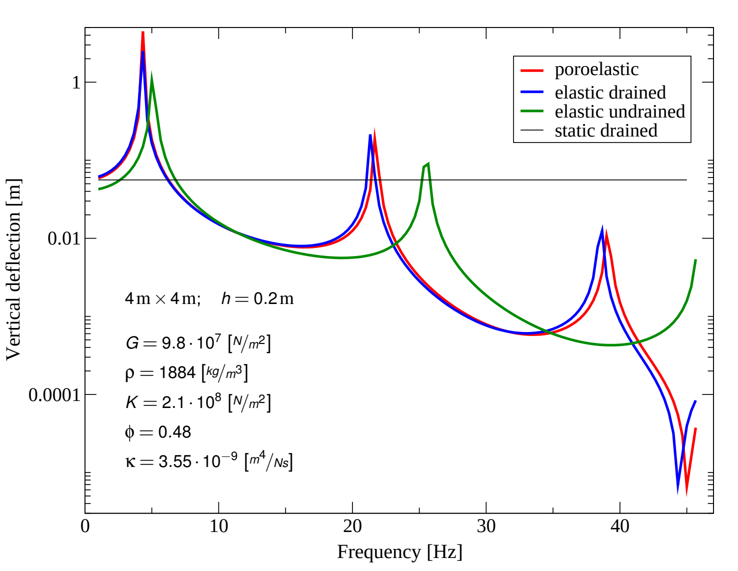
$$\int_A \Big[D[(1-\nu)\nabla\nabla w : \nabla\nabla \mathrm{v} + \nu\Delta w \Delta \mathrm{v}] - h\omega^2\rho_\beta\, w\mathrm{v} + h(B_1\, p\Delta \mathrm{v} - \beta\, p\mathrm{v}) - F\mathrm{v} \Big] \mathrm{d}A -$$

$$\int_\Gamma \left[V_n \mathrm{v} - M_{nn} \frac{\partial \mathrm{v}}{\partial n} \right] \mathrm{d}\Gamma + [M_{ns}\mathrm{v}]_{\bar{\mathbf{x}}}^{\bar{\mathbf{y}}} = 0 \qquad \bar{\mathbf{x}}, \bar{\mathbf{y}} \in \Gamma$$

$$\int_A \Big[i\omega h(B_1\Delta w\mathrm{q} - \beta\, w\mathrm{q}) - hB_2\, p\mathrm{q} - Q\mathrm{q} \Big] \mathrm{d}A = 0$$


poroelastic
elastic
static
0.01
0.0001
1e-06
Vertical deflection [m]
0
50
100
150
200
250
300
Frequency [Hz]
$4\,\mathrm{m} \times 4\,\mathrm{m}; \quad h = 0.2\,\mathrm{m}$
$G = 6.0 \cdot 10^9\ [N/m^2]$
$\rho = 2458\ [kg/m^3]$
$K = 8.0 \cdot 10^9\ [N/m^2]$
$\phi = 0.19$
$\kappa = 1.9 \cdot 10^{-10}\ [m^4/Ns]$

Numerical results



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Conclusions and Outlook

- Conclusion
  - Derivation of poroelastic plate equations by using series expansions in thickness direction
  - Different treatment of the system depending on chosen kind of truncation
  - Numerical solution of the *First Order Problem*

Conclusions and Outlook

- Conclusion
  - Derivation of poroelastic plate equations by using series expansions in thickness direction
  - Different treatment of the system depending on chosen kind of truncation
  - Numerical solution of the *First Order Problem*
- Outlook
  - Necessity to investigate higher order theories
  - Analyse the full system concerning a stable numerical solution
  - Compare the results to a 3D solution
  - Coupling the plate with an acoustic fluid

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