CRACK WIDTH CONTROL – VERIFICATION OF THE DEFORMATION COMPATIBILITY VS. COVERING THE CRACKING FORCE

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Abstract
According to the current design code the crack width control is provided by a minimum reinforcement covering any restraints as well as crack width verifications for the decisive load combinations, whereas the interaction of both is generally not pursued further. Almost all guidelines advise to control the crack width due to restraint by a minimum reinforcement taking up the cracking force respectively the cracking moment, e.g. [1] and [2]. However, such procedure neglects the deformation compatibility along the member length and the interaction of restraints and external loads is not pursued any further. Altogether, this can either lead to structural damage - mainly recognizable in form of leaking cracks in watertight constructions - or to inefficient constructions. A mechanical consistent crack width control under consideration of the real member behaviour can solely be achieved by the verification of the deformation compatibility since deformation compatibility is also the reason for restraints in the first place. Such procedure is shown for instance in [3] for the particular case of early age effects.

1. Introduction

The aim of this contribution is to establish the deformation based design method to enable efficient crack width control as well as the avoidance of expansion joints. For this purpose, important fundamentals on crack width control as well as their practical application are outlined at first. Following, the deformation based design method will be presented with special regard to structures with strict requirements on crack width but minor stressing due to external loads (usually retaining structures) as well as members with distinct tendency for cracking due to an interaction of restraint and external loads (usually large floors of jointless building constructions). The benefits of such procedure are highlighted in the conclusion.
2. Relevant fundamentals on crack width control

The crack width in reinforced concrete depends predominantly on the following parameters:

- applied strain,
- present reinforcement and its utilization (diameter, young’s modulus and steel stress),
- bond strength and bond stress distribution in the transfer length as well as
- duration of loading (short-term or permanent stress level).

The characteristically occurring crack width \( w_k \) can be derived from the difference of steel strain \( \varepsilon_s \) and concrete strain \( \varepsilon_c \) along the crack spacing \( s_r \). By simplifying the real strain distribution along \( s_r \) with the introduction of average values for steel strain \( \varepsilon_{sm} \) and concrete strain \( \varepsilon_{cm} \) the following correlation can be constituted:

\[
\begin{align*}
&w_k = \int_{x=0}^{s_r} (\varepsilon_s(x) - \varepsilon_c(x)) \, dx = s_r \cdot (\varepsilon_{sm} - \varepsilon_{cm}) \\
&\text{(1)}
\end{align*}
\]

In the crack state of single cracks (not all possible cracks have been created along the member length), the difference of steel and concrete strain occurs only in the transfer lengths to both sides of the cracks, whereas stabilized crack patterns (all possible cracks have been created along the member length) are characterized by a difference between steel and concrete strain along the whole member length. On the safe side, the crack spacing can be determined by taking into account the cracking force of the effective concrete area \( F_{cr} = A_{c,eff} \cdot f_{ct,eff} \), the average bond strength and the reinforcement diameter \( d_s \). It reads:

\[
\begin{align*}
&s_r = 2 \cdot \frac{F_{cr}}{\tau_{sm} \cdot \pi \cdot d_s} \\
&\text{(2)}
\end{align*}
\]

With respect to the influence of the load duration on the strain distribution in the crack spacing \( (k_t = 0.6 \text{ for short-term and } 0.4 \text{ for long-term}) \) and the force to be taken by the reinforcement after cracking \( F_s \) the crack width can be estimated for a given reinforcement \( (d_s, \text{ provided area } A_s, \text{ Elastic modulus } E_s) \) by a transformation of Eq. (1) and (2) in form of:

\[
\begin{align*}
&w_k = \frac{F_{cr} \cdot (F_s - k_t \cdot F_{cr}) \cdot d_s}{2 \cdot \tau_{sm} \cdot E_s \cdot A_s^2} \\
&\text{(3)}
\end{align*}
\]

As long as \( F_s \leq F_{cr} \), only single crack patterns are to be expected and \( F_s \) is to set as \( F_{cr} \) in Eq. (3). If \( F_s > F_{cr} \), all possible cracks will form along the member and a stabilized crack pattern exists. Eq. (3) considers for both crack states an undisturbed transfer length to both sides of the crack. Strictly seen, this is only correct in case of single cracks. As soon as a stabilized crack pattern has developed, the crack width might be overestimated by Eq. (3). The reason is that new cracks may form between neighbouring single cracks so that \( s_r \) decreases. Keeping in mind the transition between both crack states and statistical uncertainties of the distance between cracks in stabilized crack patterns, Eq. (3) can be seen as
justifiable for both cases. Another crucial point is the consideration of bond stresses between reinforcement and concrete which is usually simplified for conventional reinforcement with an average bond strength along the transfer lengths of $\tau_{sm} = 1.8 \cdot f_{ctm}(t)$. Further details are given e.g. in [4], [5] or [6]. Although several differences can be found in detail, Eq. (3) represents in general the EUROCODE regulations for crack width control. The direct determination of the crack width in EC2 (Sec. 7.3.4) differs only in terms of the empirical determination of the crack spacing, whereas the indirect crack width verification (Sec. 7.3.3, Table 7.2N) can be directly derived from Eq. (3). For the relation between crack width, rebar diameter and steel stress can be written according to the same assumptions in EC2 ($F_s = F_{cr\,k}$, $k_t = 0.4$, $\tau_{sm} = 1.8 \cdot f_{ctm}(t)$):

$$
\sigma_s(w_k) = \sqrt{\frac{6 \cdot w_k \cdot f_{ctm}(t) \cdot E_s}{d_s}}
$$

(4)

The EC2 regulation for the minimum reinforcement (Sec. 7.3.2) bases on the same assumptions and takes up the cracking force respectively the cracking moment. The decisive stress distribution just before cracking is considered by a factor $k_c$ (pure centric restraint $k_c = 1.0$, pure bending restraint $k_c = 0.4$), whereby the absolute size of stresses to be taken up while cracking can be modified empirically for the benefit of an efficient design. These modifications concern positively assumed influences of pre-damage due to residual stresses (factor $k$) and reduced stresses in case of early age cracking ($f_{ct,eff} < f_{ctm}$). Finally, it reads:

$$
A_{s,\text{min}} = k_c \cdot k \cdot f_{ct,eff} \cdot \frac{A_{ct}}{\sigma_s(w_k)}
$$

(5)

The application of Eq. (3) - Eq. (5) takes place in the context of a verification of the force equilibrium without further respect to the type of stressing. While this is very suitable for cases with external loads, such strategy has to be seen critically in cases with significant restraints. The main reason is that the restraint force depends strongly on the deformation compatibility and this includes also its decreasing by formation of any new crack. Furthermore, the crack pattern due to imposed deformations depends predominantly on the restraining condition which leads to a geometrically set patterns of cracks, as shown in Fig. 1. Only exception is to systems in which the steel force in the reinforcement is in complete equilibrium with the restraint force, e.g. end-restrained tension rods.

![Figure 1: Geometric set crack patterns of typical members which are predominantly restrained](image-url)
Common reinforcements have no significant influence on the occurrence of these geometric set crack patterns (primary cracks). Large quantities of reinforcement would slightly reduce $l_{cr}$, however, the steel stress in the primary cracks is not affected by the formation of a new primary crack. Thus, these primary cracks can be assumed to be independent from each other, or in other words, the geometric set cracks separate the member in parts with a length of $l_{cr}$ for which crack width control can be carried out independently.

The size of $l_{cr}$ depends predominantly on the restraining situation, whereby two principal cases can be distinguished for practical design. One is the restraining of a curvature due to self-weight activation (e.g. ground slab with temperature gradient over the height), the other one is the interaction with a rigidly connected, restraining component (e.g. shortening of a wall on a foundation). For these typical member types, [3] proposes robust engineering models to determine $l_{cr}$ on the safe side, however, a generally valid model considering the present reinforcement would require further investigations, see [7].

Besides, it needs to be said that the anchorage of the reinforcement will create secondary cracks next to the primary cracks, as shown in Fig. 2 for all cases with significant smaller effective concrete area ($A_{c,eff} < A_c$).

![Figure 2: Crack systems consisting of a primary crack and secondary cracks, here $A_{c,eff} < A_c$](image)

### 3. Crack width control on basis of deformation compatibility

#### 3.1 General Approach

Deformation compatibility is the comparison of deformation impacts in the material with the deformation response of the system. Predominant impacts are thermal expansion ($\alpha T \cdot \Delta T$), shrinkage due to hydration and drying ($\varepsilon_{cas} + \varepsilon_{cds}$) as well as creep ($\varepsilon_{cc}$), whereas the system response mainly consists of free deformation ($\Delta l / l$) and restrained deformation in form of restraint stresses ($\sigma_{rest} / E_c$). If the ratio between restraint stresses and real length change is expressed by a so-called restraint degree $a$, it can be written in the uncracked state:

$$ (\alpha T \cdot \Delta T + \varepsilon_{cas} + \varepsilon_{cds} + \varepsilon_{cc}) \cdot a = - \frac{\sigma_{rest}}{E_c} $$

(6)
If restraint stresses exceed a certain limit value of the tensile strength, cracking is to be expected. After cracking the deformation compatibility has to take into account the cracks with their certain width as well. As explained before, the geometric set cracks separate the member in independent parts with a length of $l_{cr}$, so that the deformation compatibility can be verified representatively for one primary crack in the length of $l_{cr}$. Besides, the stiffness of the restrained member decreases due to cracking, so that the restraint degree increases in all cases where the restraining condition is the same after cracking. With regard to the restrained deformation in the transfer length, it can be written for the cracked state:

$$
(\alpha_T \cdot \Delta T + \varepsilon_{cas} + \varepsilon_{cds} + \varepsilon_{cc}) \cdot a^{II} \cdot l_{cr} = -\left[\frac{\sigma_{rest}}{E_c} \cdot l_{cr} + s_r \cdot (1 - k_t) \cdot \left(\varepsilon_{sm} - \frac{\sigma_{rest}^{II}}{E_c}\right)\right]
$$

(7)

And on basis of the equilibrium between steel force in the primary crack and concrete force in the uncracked part between the primary cracks, the crack width amounts:

$$
w = -(\alpha_T \cdot \Delta T + \varepsilon_{cas} + \varepsilon_{cds} + \varepsilon_{cc}) \cdot a^{II} \cdot l_{cr} - \frac{\sigma_s \cdot A_s}{E_c \cdot A_c} \cdot (l_{cr} - (1 - k_t) \cdot s_r \cdot (1 - k_t))
$$

(8)

Finally, the required reinforcement to limit the crack width under consideration of the deformation compatibility amounts:

$$
A_s = \frac{-(\alpha_T \cdot \Delta T + \varepsilon_{cas} + \varepsilon_{cds} + \varepsilon_{cc}) \cdot a^{II} \cdot l_{cr} - w_k \cdot E_c \cdot A_c}{(l_{cr} - (1 - k_t) \cdot s_r \cdot (1 - k_t)) \cdot \sigma_s(w_k)}
$$

(9)

The solution of Eq. (9) is not trivial, the following are the main challenges:

- determination of the restraint degree after cracking, in particular if present reinforcement has significant influence,
- consideration of remaining concrete stresses in the uncracked part between two primary cracks, especially if these stresses vary over the height of the cross section and along the member length as in case of bottom-restrained members (e.g. walls on foundations) and
- consideration of secondary cracking.

A possible simplification is that the primary crack or the crack system consisting of primary crack and secondary cracks will have to absorb the entire restrained deformation of the uncracked state. In practical cases, this assumption is conservative because the restrained deformation in the concrete between primary cracks after cracking is neglected, even though it is bigger than the influence of any possible increase of the restraint degree after cracking. But it is also appropriate because demanding iterations due to the above listed points can be avoided. It reads:

$$
w^p + \sum_{i=1}^{n} w_i^s = \frac{\sigma_{rest}}{E_c} \cdot l_{cr}
$$

(10)
The basic idea of Eq. (10) is to create as many secondary cracks in the surrounding of the primary crack as needed to limit the crack width in the primary crack. The required number of secondary cracks can be determined with regard to the crack width criteria $w_k$ and the simplification behind Eq. (10) by:

$$n = \left( \frac{\sigma_{\text{rest}}}{E_c} \cdot l_{er} \cdot \frac{1}{w_k} - 1 \right) \cdot 1.1 \quad (11)$$

The decreasing width of subsequently occurring secondary cracks in comparison to the width of the primary crack is expressed by the factor 1.1 which covers relevant practical situations.

The minimum reinforcement required can be derived from the number of secondary cracks $n$, where $n$ is rounded up to the next integer. If $n \leq 0$ the deformation compatibility is already fulfilled with the width of the primary crack and reinforcement for crack width control is not needed. Only a skin reinforcement taking up the cracking force of the effective concrete area would be recommended. All other cases require active crack width control and the required minimum reinforcement can be determined according to [8]. Altogether, it reads:

$$n \leq 0: \quad A_{s,\text{req}} = \frac{f_{\text{ctm}}}{f_{yk}} \cdot A_{c,\text{eff}} \quad (12)$$

with:
- $f_{\text{ctm}}$ number of secondary cracks required
- $f_{yk}$ yield strength of reinforcement
- $A_{c,\text{eff}}$ effective concrete area (normally $2.5 \cdot d_1 \cdot b$)

$$n > 0: \quad A_{s,\text{req}} = \frac{d_s \cdot b^2 \cdot d_1^2 \cdot f_{\text{ct,eff}} \cdot (0.69 + 0.34 \cdot n)}{w_k \cdot E_s} \quad (13)$$

with:
- $d_s$ reinforcement diameter
- $b$ width in direction viewed (normally 1 m)
- $d_1$ edge-distance of the reinforcement
- $f_{\text{ct,eff}}$ decisive tensile strength of concrete
- $w_k$ crack width criteria
- $E_s$ elastic modulus of reinforcement

### 3.1 Application for members which are primarily restrained

In the majority of primarily restrained members, the quantification of restraint stresses respectively the restrained deformation to be absorbed requires a time discrete analysis. On the one hand, the deformation impacts appear usually by time, whereas an enormous part already occurs in the hardening phase interacting with considerable stiffness changes of the concrete. On the other hand, the size and course of time of creep deformations strongly depends on the stress history and can hardly be predicted a priori. To quantify the restrained deformation various analytical, semi-numerical or numerical approaches are available in the literature. The application of these models still require expertise beyond practical engineering knowledge. Implemented material models and their calibration to the behaviour of the
concrete used as well as considered boundary conditions (thermal and mechanical ones) can have a significant influence on the determined restrained and should be verified carefully. Some minimum standards of the latter are shown in Fig. 3 for the practical cases.

If the restrained deformation is quantified and cracking cannot be excluded, the required minimum reinforcement can be determined with Eq. (11) - Eq. (13). It should be mentioned that residual stresses should be removed from the decisive stress distribution before applying Eq. (11). The reason is that residual stresses or so-called *Eigenstresses* are self-balanced within the cross section and will usually cause solely microcracks or small, locally restricted cracks. Of course, this pre-damage can be the starting point of a macrocrack, however, residual stresses decrease considerably in the cross section by this time and have no relevant contribution to the opening of the macrocrack. Fig. 4 illustrates these thoughts.

![Diagram](image)

**Figure 3:** Volumetric idealizations of typical concrete members which are primarily restrained

![Diagram](image)

**Figure 4:** Role of residual stresses or so-called *Eigenstresses* on the process of macrocracking

Application examples of this approach are given in [3] and [9]. Further considerations with focus on hardening-induced stresses in very thick concrete members were presented in [10].

### 3.2 Application for members with significant interaction of restraint and loads

In contrast to members which are primarily restrained and which show therefore a geometric set crack pattern, the application in case of significant interaction of restraints and loads has to consider members with areas being considerably stressed respectively already showing distinct crack patterns. An illustrative example is jointless floors which are end-restrained by building cores and/or retaining walls. On the one hand, such floors are permanently subjected to bending due to loads (self-weight as well as service loads) and on the other hand, imposed deformations (early age effects as well as drying shrinkage and temperature changes during
service life) will cause a restraint force due to interaction with the restraining condition, as illustrated in Fig. 5. The size of the occurring restraint force $N_{\text{rest}}$ depends not only on the stiffness of the restraining condition but also on the released deformation in the cracks. This means, the deformation compatibility can be complied by the elongation of the member in the cracked areas as well as the restrained deformation in the uncracked parts.

![Figure 5: Interaction of restraint and loads in case of jointless floors between cores](image)

Taking into account the imposed and remaining deformation due to hardening ($\varepsilon_0$), drying shrinkage and affine creep ($\varepsilon_{\text{cds}} + \varepsilon_{\text{cc}}$) as well as temperature induced shortening during service live ($\alpha_T \cdot \Delta T_N$) the deformation compatibility can be complied according to Eq. (1) and Eq. (7). In case of bending, as in the present example, it is also important to consider the decrease of released deformation between opening of the bending crack and elongation of the member itself. By simplifying this context in the first step with a conservative assumption of a plane section in the bending crack but neglecting the concrete strain in the compression zone, this effect can be represented by a factor of 0.5. Finally it reads:

$$[\varepsilon_0 + (\varepsilon_{\text{cds}} + \varepsilon_{\text{cc}} + \alpha_T \cdot \Delta T_N) \cdot a] \cdot l = - \left[ \frac{\sigma_{\text{cm}}}{E_c} \cdot (l - l^{III}) + 0.5 \cdot \varepsilon_{\text{sm}} \cdot l^{III} \right]$$

Hereby, $l$ stands for the member length (in case of the floors it would be one field) and $l^{III}$ represents the length with a distinct crack pattern within $l$. With regard to the steel stress limitation under permanent loads, which may not assure a stabilized crack pattern over the full length, the required length of the floor to be in a cracked state can be determined following Eq. (14) by:

$$l^{III}_{\text{req}} = \frac{[\varepsilon_0 + (\varepsilon_{\text{cds}} + \varepsilon_{\text{cc}} + \alpha_T \cdot \Delta T_N) \cdot a + \frac{\sigma_{\text{cm}}}{E_c}]}{\frac{\sigma_{\text{cm}}}{E_c} - 0.5 \cdot (1 - k_\ell) \cdot \frac{\sigma_s(w_N)}{E_s}} \cdot l$$

For the solution of Eq. (15), the average concrete stress in the uncracked part ($\sigma_{\text{c,m}}$) has to be determined iteratively. In detail, all cases where the required length to be cracked is already smaller than the cracked length due to permanent loads, no restraints occur at all and $\sigma_{\text{c,m}}$ is zero anyway. In all other cases, $\sigma_{\text{c,m}}$ will increase up to a size with which the superposition of bending stresses due to permanent loads and $\sigma_{\text{c,m}}$ will have created a sufficient long cracked
length. The hereby occurring size of $\sigma_{c,m}$ can be determined with Eq. (16) whereas the location of the decisive moment under permanent loads is illustrated in Fig. 6.

$$\sigma_{c,m} = \max \left\{ 0; f_{ctm} - \frac{M_{\text{perm}}(x^H)}{W} \right\}$$  \hspace{1cm} (16)

Figure 6: Interaction of restraint and loads in case of jointless floors between cores

Finally, the required reinforcement can be determined by a transformation of Eq. (3) in form of:

$$A_{s,\text{req}} = \sqrt{\frac{F_{cr} \cdot (F_s - k_t \cdot F_{cr}) \cdot d_s}{2 \cdot \tau_{sm} \cdot E_s \cdot w_k}}$$  \hspace{1cm} (17)

The force to be taken by the reinforcement ($F_s$) results hereby from the moment under permanent loads as well as from the restraint force with a size of:

$$N_{\text{rest}} = \sigma_{c,m} \cdot A_c$$  \hspace{1cm} (18)

The application of this approach in several building constructions, e.g. "Konzernzentrale Premiere München-Unterföhring" with an irregularly shaped 180 m long jointless floor constrained by several very stiff cores or "Highlight Towers Parkstadt Schwabing" with 33 levels of 80 m long floors constrained by two cores, showed, that this enables a very efficient design under avoidance of expansion joints. The most important point is hereby, that the restraint force under consideration of the stresses due to loading is much smaller than the usually considered cracking force. This provides not only efficient reinforcement amounts in the floors but also an appropriate consideration of the interaction between the floors and the building cores.

Figure 7: Interaction of restraint and loads in case of jointless floors between cores
4. Discussion and conclusion

It is standard practice to determine the required minimum reinforcement for crack width control on basis of the cracking force respectively the cracking moment. However, such procedure neglects the deformation compatibility along the member length and the interaction of restraints and external loads is not considered appropriately. Altogether, this can lead to structural damage in form of leaking cracks in watertight constructions or to inefficiency.

This contribution presents fundamentals on a new design concept which bases on the verification of the deformation compatibility. The biggest improvement of this approach is the realistic consideration of the occurring restraints with respect to its decreasing by formation of any new crack. The approach can be applied for primarily restrained members as well as for members with significant interaction of restraint and loads. By taking into account the real deformation impact on the member as well as the member dimensions, the empiricism of minimum reinforcement design for crack width control due to restraint is reduced significantly. Furthermore, the flow of forces in constructions with significant interaction of restraint and loads can be taken into account realistically. Besides the benefits of mechanical proof, this concept also provides an opportunity for direct cooperation between concrete technology, structural design and construction site, offering an important contribution to the crack width control of concrete members.

References