Towards iterative 2D and 3D kinetic modelling of RMP interaction with tokamak plasmas

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Suppressing edge-localized modes (ELMs) with help of external resonant magnetic field perturbations (RMPs) is an experimentally proven technique for the reduction of tokamak first wall damage by large, type-I ELMs. Linear and nonlinear modelling of the interaction of RMPs with tokamak plasma is presently developing on the basis of MHD and kinetic theory [1]. In this report, we extend an iterative approach proposed in Ref. [2] and realized in the code MEPHIT for the ideal MHD plasma response in Ref. [3] to the approach combining the ideal MHD and kinetic plasma response. In this approach, plasma response current density computed for the given perturbation magnetic field and computations of this field by a finite element Maxwell solver in realistic tokamak geometry with a given response current coupled together within an iterative procedure.

Plasma response in cylindrical geometry

In the inhomogeneous straight cylinder geometry with identical ends, the perturbation of charge and parallel current density due to plasma response are computed from the analytical solution of the gyrokinetic equation with an Ornstein-Uhlenbeck collision operator with an energy preserving term [1]. Up to the leading order in Larmor radius, Fourier amplitudes of these quantities expanded over the poloidal angle ϑ and the toroidal angle $\varphi = z/R_0$ with harmonic index $\mathbf{m} = (m,n)$ are

$$\begin{aligned}
\rho_{\mathbf{m}} &= \frac{k_\parallel}{\omega_E} F_{\mathbf{m}} \left(\frac{B^r_{\mathbf{m}}}{B_0} - \frac{i k_\parallel \Phi_{\mathbf{m}}}{E_{0r}} \right) + \frac{\partial^2}{\partial r^2} \frac{k_\parallel}{\omega_E} G_{\mathbf{m}} \left(\frac{B^r_{\mathbf{m}}}{B_0} - \frac{i k_\parallel \Phi_{\mathbf{m}}}{E_{0r}} \right) \\
&+ \frac{k_\parallel}{\omega_E} G_{\mathbf{m}} \left(\frac{\partial^2}{\partial r^2} \frac{B^r_{\mathbf{m}}}{B_0} - \frac{i k_\parallel}{E_{0r}} \frac{\partial^2}{\partial r^2} \Phi_{\mathbf{m}} \right) + \frac{\partial^2}{\partial r^2} H \Phi_{\mathbf{m}} + H \frac{\partial^2}{\partial r^2} \Phi_{\mathbf{m}}, \qquad (1)
\end{aligned}$$

$$\begin{aligned}
j_{\parallel \mathbf{m}} &= -F_{\mathbf{m}} \left(\frac{B^r_{\mathbf{m}}}{B_0} - \frac{i k_\parallel \Phi_{\mathbf{m}}}{E_{0r}} \right) - \frac{\partial^2}{\partial r^2} G_{\mathbf{m}} \left(\frac{B^r_{\mathbf{m}}}{B_0} - \frac{i k_\parallel \Phi_{\mathbf{m}}}{E_{0r}} \right) \\
&- G_{\mathbf{m}} \left(\frac{\partial^2}{\partial r^2} \frac{B^r_{\mathbf{m}}}{B_0} - \frac{i k_\parallel}{E_{0r}} \frac{\partial^2}{\partial r^2} \Phi_{\mathbf{m}} \right). \qquad (2)
\end{aligned}$$

Here, (r, ϑ, z) are cylindrical coordinates, R_0 is the major radius, $B^r_{\mathbf{m}}$ and $\Phi_{\mathbf{m}}$ are Fourier amplitudes of the radial magnetic field and electrostatic potential of the perturbation, B_0 and E_{0r} are the equilibrium magnetic and radial electric field, $\omega_E = k_\perp V_E$ where V_E is the equilibrium $E \times B$ rotation velocity, $k_\parallel = (m+nq)/(qR_0)$ and $k_\perp \approx m/r$ are parallel and perpendicular wave-vectors, respectively, with q being the safety factor. The coefficients,

$$F_{\mathbf{m}} = \sum_\alpha \frac{e_\alpha n_\alpha v_{T\alpha}^2}{\nu_\alpha}\left((A_1^\alpha + A_2^\alpha) I^{11} + \frac{1}{2} A_2^\alpha I^{13}\right), \tag{3}$$

$$G_{\mathbf{m}} = \sum_\alpha \frac{e_\alpha n_\alpha v_{T\alpha}^2 \rho_{L\alpha}^2}{2\nu_\alpha}\left((A_1^\alpha + 2A_2^\alpha) I^{11} + \frac{1}{2} A_2^\alpha I^{13}\right), \tag{4}$$

$$H = -\sum_\alpha \frac{e_\alpha n_\alpha \rho_{L\alpha}^2}{2E_{0r}}\left(A_1^\alpha + \frac{5}{2} A_2^\alpha\right) = \frac{1}{8\pi}\sum_\alpha \frac{\rho_{L\alpha}^2 V_{\alpha\perp}}{r_{D\alpha}^2 V_E}, \tag{5}$$

are expressed via thermodynamic forces of species α,

$$A_1^\alpha = \frac{1}{n_\alpha}\frac{\partial n_\alpha}{\partial r} - \frac{e_\alpha E_{0r}}{T_\alpha} - \frac{3}{2T_\alpha}\frac{\partial T_\alpha}{\partial r}, \qquad A_2^\alpha = \frac{1}{T_\alpha}\frac{\partial T_\alpha}{\partial r}, \tag{6}$$

and the dimensionless moments of the Green's function of the kinetic equation I^{jk} (susceptibility functions, see Refs. [1, 5]) with e_α, n_α, T_α, $v_{T\alpha}$, $\rho_{L\alpha}$, ν_α, $r_{D\alpha}$ and $V_{\alpha\perp}$ being charge, density, temperature, thermal velocity, Larmor radius, collision frequency, Debye radius and perpendicular fluid velocity of species α, respectively. It can be seen that the response of zero order in Larmor radius is proportional to $F_{\mathbf{m}}$ and is solely driven by the combination

$$\frac{B^r_{\mathbf{m}}}{B_0} - \frac{ik_\parallel \Phi_{\mathbf{m}}}{E_{0r}} = \frac{E^{MA}_{\parallel\mathbf{m}}}{E_{0r}} = \frac{k_\parallel E^{MA}_{\perp\mathbf{m}}}{k_\perp E_{0r}}, \tag{7}$$

which is proportional to the parallel component of the total electric field within perturbed flux surfaces, $E^{MA}_{\parallel\mathbf{m}}$, or alternatively, the perpendicular component $E^{MA}_{\perp\mathbf{m}}$, which come from the mis-alignment of perturbed equi-potential and magnetic flux surfaces [5].

Perturbations of charge and current density determine perturbations of the potential and magnetic field via Laplace's equation and Ampere's law. It should be noted that the trend of $\Phi_{\mathbf{m}}$ is to annihilate the mis-alignment field due to flux surface meandering (first term in Eq. (7)) in the ideal regions outside resonant layers centered around the rational flux surfaces where the so-called "external" solution is well described by ideal MHD and where the mis-alignment field is exponentially small. Since the parallel current density (2) does not include contributions from cyclotron harmonics, this current is strongly localized in the resonant layers while the ideal equilibrium currents important for the outer solution are fully missing (intentionally) and should be handled separately.

Extension to the toroidal geometry

The cylindrical plasma model discussed above is used here to set up the combined kinetic–ideal MHD approach within the iteration scheme of the code MEPHIT [3] providing a correct (by an order of magnitude) value of the plasma response current which will later be replaced with a fully 3D evaluation by the code GORILLA [4]. For that, we note that magnetic field reconnection as well as ideal MHD shielding currents are driven solely by Fourier harmonics of the ratio B^r/B_0^φ where B_0^φ is the contra-variant toroidal component of the equilibrium magnetic field. In ideal MHD, each of these harmonics induces a current sheet at the resonant surface with a divergence-free current density described by a single harmonic (with the same index $\mathbf{m}$) of the ratio $j_\parallel/B_0$. Ignoring the harmonic coupling, we can extend Eqs. (1) to toroidal geometry where Laplace's equation takes the form

$$\begin{aligned}
\frac{\partial^2\Phi_\mathbf{m}}{\partial r^2} &= -\frac{4\pi(m+nq)}{q\omega_E\overline{R^2|\nabla r|^2}}F_\mathbf{m}\left(\left(\frac{B^r}{B_0^\varphi}\right)_\mathbf{m} - i\frac{m+nq}{qE_{0r}}\Phi_\mathbf{m}\right) \\
&- \frac{\partial^2}{\partial r^2}\frac{4\pi(m+nq)}{q\omega_E}G_\mathbf{m}^{\text{tor}}\left(\left(\frac{B^r}{B_0^\varphi}\right)_\mathbf{m} - i\frac{m+nq}{qE_{0r}}\Phi_\mathbf{m}\right) \\
&- \frac{4\pi(m+nq)}{q\omega_E}G_\mathbf{m}^{\text{tor}}\left(\frac{\partial^2}{\partial r^2}\left(\frac{B^r}{B_0^\varphi}\right)_\mathbf{m} - i\frac{m+nq}{qE_{0r}}\frac{\partial^2}{\partial r^2}\Phi_\mathbf{m}\right) \\
&- 4\pi\left(\frac{\partial^2}{\partial r^2}H^{\text{tor}}\Phi_\mathbf{m} + H^{\text{tor}}\frac{\partial^2}{\partial r^2}\Phi_\mathbf{m}\right). \qquad (8)
\end{aligned}$$

Here, $G_\mathbf{m}^{\text{tor}} = G_\mathbf{m}B_0^2B_{0\varphi}^{-2}$ and $H^{\text{tor}} = HB_0^2R_0^2B_{0\varphi}^{-2}$ contain only flux functions in the pre-factor, and $\overline{R^2|\nabla r|^2}$ means averaging over the angles (ϑ, φ) of the straight field line flux coordinate system with r being some flux surface label (poloidal flux in our case). Eq. (8) is a second-order ODE in the radial variable with mode coupling ignored because mode coupling introduced by the metric tensor is weak for narrow resonant layers where the dependence on radius dominates. Eq. (8) is solved by a lowest-order finite difference scheme assuming fixed magnetic field perturbation. The result is used then for computation of the Fourier amplitude of the parallel current density perturbation

$$\begin{aligned}
\left(\frac{j_\parallel}{B_0}\right)_\mathbf{m} &= -\frac{F_\mathbf{m}}{B_{0\varphi}}\left(\left(\frac{B^r}{B_0^\varphi}\right)_\mathbf{m} - i\frac{m+nq}{qE_{0r}}\Phi_\mathbf{m}\right) \\
&- \frac{\partial^2}{\partial r^2}\overline{R^2|\nabla r|^2}\frac{G_\mathbf{m}^{\text{tor}}}{B_{0\varphi}}\left(\left(\frac{B^r}{B_0^\varphi}\right)_\mathbf{m} - i\frac{m+nq}{qE_{0r}}\Phi_\mathbf{m}\right) \\
&- \overline{R^2|\nabla r|^2}\frac{G_\mathbf{m}^{\text{tor}}}{B_{0\varphi}}\left(\frac{\partial^2}{\partial r^2}\left(\frac{B^r}{B_0^\varphi}\right)_\mathbf{m} - i\frac{m+nq}{qE_{0r}}\frac{\partial^2}{\partial r^2}\Phi_\mathbf{m}\right), \qquad (9)
\end{aligned}$$

which follows from extension of Eq. (2). The current density (9) is strongly localized within the resonant layer whose width [5] is determined by the electron component in a typical collisional regime as $\delta_\mathbf{m} \sim |k_\parallel' v_{Te}|^{-1}\sqrt{\nu_e|\omega_E|}$ where $k_\parallel'$ means the derivative over the surface label. At the distance $\delta_\mathbf{m}$ and further away from the resonant surface, the ideal MHD current density, which is singular at the resonant surface and scales with $k_{\parallel}^{-2}$ in the presence of a pressure gradient, scales to the kinetic response current density (9) as plasma β or smaller. Therefore, we introduce in the vicinity $\delta_{\mathbf{m}}$ of the resonant surface an artificial damping in magnetic differential equations (16) and (19) of Ref. [3] when computing the ideal MHD response current by adding to the toroidal mode number n a small imaginary part $inq'(r)/q\sum_{\mathbf{m}}\delta_{\mathbf{m}}\exp\left(-(r-r_{\mathbf{m}})^2/\delta_{\mathbf{m}}^2\right)$ with safety factor q and resonance position $r_{\mathbf{m}}$. Thus, we remove the unphysical singularity of the ideal MHD current coming from the assumption of pressure being constant on perturbed flux surfaces, which does not hold in the resonant layer. Combination of the resulting ideal MHD currents and kinetic current (9) computed for fixed perturbation magnetic field is used then within MEPHIT iteration scheme to obtain self-consistent currents and the magnetic field, as seen in Fig. 1.

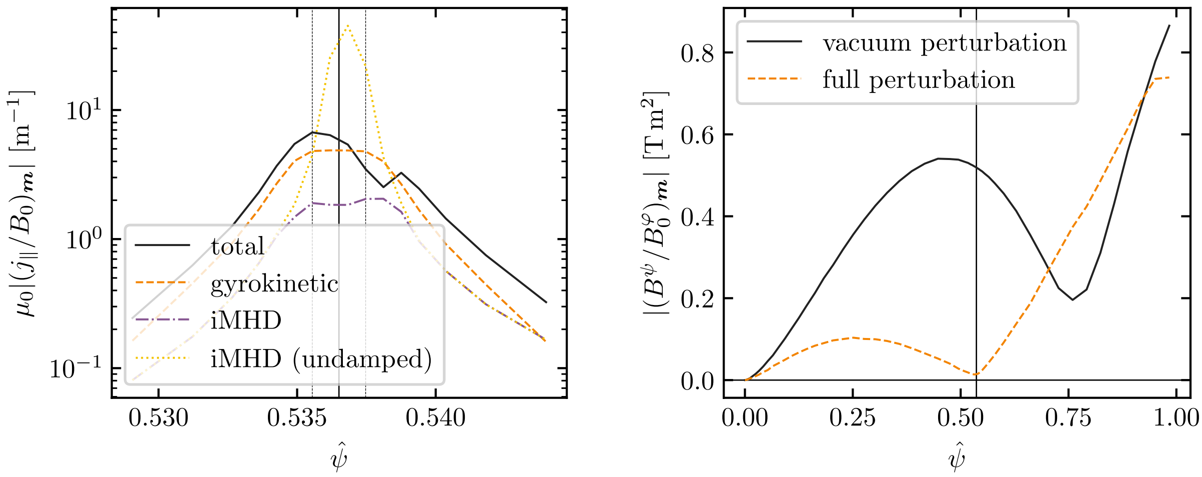

Figure 1: Initial parallel plasma response current density (left) and final radial magnetic field perturbation (right). $\hat{\psi}$ is the normalized poloidal flux. Solid vertical lines indicate the resonance position for $\mathbf{m} = (3,2)$ and dashed vertical lines delimit the corresponding resonant layer.

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