

SAT-Based Methods for Circuit Synthesis

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Abstract—Reactive synthesis supports designers by automatically constructing correct hardware from declarative specifications. Synthesis algorithms usually compute a strategy, and then construct a circuit that implements it. In this work, we study SAT- and QBF-based methods for the second step, i.e., computing circuits from strategies. This includes methods based on QBF-certification, interpolation, and computational learning. We present optimizations, efficient implementations, and experimental results for synthesis from safety specifications, where we outperform BDDs both regarding execution time and circuit size. This is an extended version of [2], with an additional appendix.

I. INTRODUCTION

Synthesis is an ambitious design approach: Instead of checking whether an already constructed system satisfies its specification, a correct implementation is derived *automatically* from the specification [3]. Synthesis is also used in rapid prototyping, automatic repair [9], and program sketching [14].

Existing work often focuses on finding strategies to satisfy the specification, or only on deciding realizability. However, computing circuits from strategies is computationally demanding as well. System quality (e.g., circuit size and depth) imposes additional challenges. Synthesized strategies usually allow for much implementation freedom, which needs to be exploited cleverly. Algorithms must also be symbolic (operate on formulas rather than enumerating states) to achieve scalability. These symbolic algorithms are usually implemented with BDDs because they offer existential *and* universal quantification. Recently, SAT-based synthesis algorithms have been proposed [12], [4] as alternative to BDDs and their scalability issues. However, these works do not address circuit extraction.

We thus present and compare several SAT- and QBF-based circuit synthesis algorithms. The basic algorithms are not new, but we present novel optimizations, combinations, efficient implementations for safety synthesis problems, and extensive experiments. This includes methods based on QBF-certification, computational learning (including the first application of incremental QBF solving in synthesis), and interpolation. We achieve the best results by combining ideas from interpolation [8] with learning [7], thereby outperforming BDDs both regarding computation time and circuit size.

Related work. It is argued [7] that many circuit synthesis methods are still outperformed by the simple BDD-based cofactor approach [3]. The same work [7] also proposes learning-based techniques, which are implemented with BDDs. This

yields smaller circuits, but is slower. We show how learning can be efficiently realized with SAT- and QBF-solvers, and that this realization can outperform the cofactor approach both regarding circuit size and computation time. SAT-based learning is also used in [4]. However, this work only addresses strategy computation and not circuit synthesis. Jiang et al. [8] propose interpolation for circuit extraction, and show how quantifier alternations can be avoided by temporarily treating outputs as inputs. We combine this idea with learning to compute interpolants, thereby achieving a speedup of several orders of magnitude. QBF certification [13] can derive circuits from a completeness proof of the strategy formula. We show how this method can be applied efficiently for safety synthesis.

II. PRELIMINARIES

We assume familiarity with propositional logic, SAT- and QBF-solving (cf. [1]) but repeat the most important concepts.

Basic Notation. A *literal* is a Boolean variable or its negation. A *clause (cube)* is a disjunction (conjunction) of literals, and a *Conjunctive Normal Form (CNF)* formula is a conjunction of clauses. We denote variables vectors with overlines, corresponding cubes in bold, and propositional formulas with capital letters. E.g., \mathbf{x} is a cube over the variable vector $\bar{x} = (x_1, \dots, x_n)$, and $F(\bar{x})$ is a formula over \bar{x} . If the variables are irrelevant, we simply write F instead of $F(\bar{x})$.

Decision Procedures. A *SAT-solver* checks if a CNF is satisfiable. We write $(\text{sat}, \mathbf{x}) := \text{PSAT}(F(\bar{x}))$ for a SAT-solver call, where sat is assigned **true** iff the CNF F is satisfiable, and \mathbf{x} is a satisfying assignment given as cube over \bar{x} . Let \mathbf{x} be a cube. We write $\mathbf{y} := \text{PCORE}(\mathbf{x}, F)$ to denote the extraction of an unsatisfiable core: Given that $\mathbf{x} \wedge F$ is unsatisfiable, \mathbf{y} will be a sub-cube of \mathbf{x} such that $\mathbf{y} \wedge F$ is still unsatisfiable. Let $A(\bar{x}, \bar{y})$ and $B(\bar{x}, \bar{z})$ be two CNFs such that $A \wedge B$ is unsatisfiable, and \bar{y} and \bar{z} are disjoint. An *interpolant* is a formula $I(\bar{x})$ such that $A \Rightarrow I \Rightarrow \neg B$. Interpolants can be computed from the unsatisfiability proof of $A \wedge B$ [6]. We denote this computation by $I := \text{INT}(A, B)$. A *Quantified Boolean Formula (QBF)* is a formula $Q_1 \bar{x} . Q_2 \bar{y} . \dots F(\bar{x}, \bar{y}, \dots)$, where $Q_i \in \{\forall, \exists\}$ and F is a CNF. The quantifiers have their expected semantics. A *QBF-solver* checks if a QBF is satisfiable. We write $(\text{sat}, \mathbf{a}) := \text{QSAT}(\exists \bar{a} . Q_1 \bar{x} . Q_2 \bar{y} . \dots F(\bar{a}, \bar{x}, \bar{y}, \dots))$ for QBF-solver calls. The satisfying assignment \mathbf{a} can only be extracted for variables that are quantified existentially on the outermost level. Finally, we write $\mathbf{b} := \text{QCORE}(\mathbf{a}, \exists \bar{a} . Q_1 \bar{x} . Q_2 \bar{y} . \dots F(\bar{a}, \bar{x}, \bar{y}, \dots))$ to denote the extraction of an unsatisfiable core.

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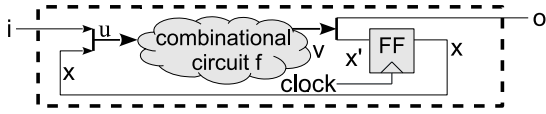


Fig. 1. Implementation of a strategy. (FF = flip-flops).

Circuit Synthesis. A *strategy* is a formula $S(\bar{x}, \bar{i}, \bar{o}, \bar{x}')$ such that $\forall \bar{x}, \bar{i}. \exists \bar{o}, \bar{x}'. S$, where $\bar{x}, \bar{i}, \bar{o}$ are state-, input-, and output-bits, respectively, and \bar{x}' is the next-state copy of \bar{x} . Intuitively, for a given state \mathbf{x} and input \mathbf{i} , S defines allowed output-values \mathbf{o} and next states \mathbf{x}' : \mathbf{o}, \mathbf{x}' is *allowed* iff $\mathbf{x} \wedge \mathbf{i} \wedge \mathbf{o} \wedge \mathbf{x}'$ satisfies S . Let $\bar{u} = \bar{x} \cup \bar{i}$ and $\bar{v} = \bar{o} \cup \bar{x}'$. An *implementation* of $S(\bar{u}, \bar{v})$ is a function $f : 2^{|\bar{u}|} \rightarrow 2^{|\bar{v}|}$ such that $\forall \bar{u}. S(\bar{u}, f(\bar{u}))$. This function can be implemented in hardware as shown in Fig. 1.

Strategies for safety specifications are particularly simple: given a *winning region* $W(\bar{x})$ from which the specification can be enforced, and a complete¹ and deterministic² transition relation $T(\bar{x}, \bar{i}, \bar{o}, \bar{x}')$ defining the state transitions, the strategy must pick values for \bar{o} such that the next state is in W again, i.e., $S = (\neg W(\bar{x})) \vee (T(\bar{x}, \bar{i}, \bar{o}, \bar{x}') \wedge W(\bar{x}'))$. We only need to synthesize circuits for \bar{o} , and define \bar{x}' using T .

III. CIRCUIT SYNTHESIS ALGORITHMS

A. QBF-Certification

An implementation can be computed as Skolem functions³ for the signals \bar{o} and \bar{x}' in the QBF $\forall \bar{x}, \bar{i}. \exists \bar{o}, \bar{x}'. S(\bar{x}, \bar{i}, \bar{o}, \bar{x}')$. QBF Cert [13] computes such functions using DepQBF [10].

Optimizations for Safety Specifications. We need to find Skolem functions for \bar{o} in $\forall \bar{x}, \bar{i}. \exists \bar{o}, \bar{x}'. (\neg W) \vee (T \wedge W')$. Yet, we achieve better results with QBF Cert by computing Herbrand functions⁴ in the unsatisfiable QBF $\exists \bar{x}, \bar{i}. \forall \bar{o}. \exists \bar{x}'. W \wedge T \wedge \neg W'$. This works because T is deterministic and complete. In our setting, W is in CNF, so the conjunctions in the latter formulation are simpler to realize in CNF. Also, the clause resolution proofs required for unsatisfiable QBFs are usually less expensive than the cube resolution proofs for satisfiable ones. Still, the intermediate files produced by QBF Cert can grow large (hundreds of GB). One reason is that a straightforward CNF encoding of $\neg W'$ requires many auxiliary variables and clauses. We could reduce the size of the files by up to a factor of 30 by learning a CNF representation of $\neg W'$ without introducing auxiliary variables using the following algorithm:

- 1: **procedure** NEGLEARN(W'), **returns:** $\neg W'$
- 2: $N' := \mathbf{true}$
- 3: **while** sat, with (sat, \mathbf{x}) := PSAT($W' \wedge N'$) **do**
- 4: $N' := N' \wedge \neg \text{PCORE}(\mathbf{x}, \neg W')$
- 5: **return** N'

¹I.e., $\forall \bar{x}, \bar{i}, \bar{o}. \exists \bar{x}'. T(\bar{x}, \bar{i}, \bar{o}, \bar{x}')$. T can always be made complete: if some input is not allowed by the original specification, T can allow for arbitrary outputs; if some output is not allowed originally, T can visit an unsafe state.

²I.e., $\forall \bar{x}, \bar{i}, \bar{o}, \bar{x}_1', \bar{x}_2'. (T(\bar{x}, \bar{i}, \bar{o}, \bar{x}_1') \wedge T(\bar{x}, \bar{i}, \bar{o}, \bar{x}_2')) \Rightarrow (\bar{x}_1' = \bar{x}_2')$.

³Skolem functions define existentially quantified variables as a function over the universally quantified ones such that the QBF becomes **true**.

⁴Herbrand functions define universally quantified variables as a function over the existentially quantified ones such that the QBF becomes **false**.

As long as N' is not yet $\neg W'$, i.e., $W' \wedge N'$ is still satisfiable, we refine N' with a clause that excludes the cube \mathbf{x} witnessing this insufficiency. By taking the unsatisfiable core, the clause eliminates also other counterexamples. Since clauses are only added, NEGLEARN is suitable for incremental SAT solving.

Using incremental SAT solving, we also simplify W by removing literals and clauses as long as W does not change semantically. This is applied to all following methods as well.

B. QBF-Based Learning

In [7], several learning-based circuit synthesis algorithms are presented and implemented using BDDs. Here, we discuss an efficient implementation of the CNF-learning algorithm using a QBF-solver. Since QBF-solvers operate on CNFs, this algorithm is particularly suitable. It can be defined as follows.

- 1: **procedure** SYLEARNQBF($S(\bar{x}, \bar{i}, \bar{o}, \bar{x}')$)
- 2: $\bar{u} := \bar{x} \cup \bar{i}, \bar{v}_a := \bar{v} := \bar{o} \cup \bar{x}'$
- 3: **for** $v \in \bar{v}$ **do**
- 4: $\bar{v}_a := \bar{v}_a \setminus \{v\}, \bar{v}_e := \bar{v} \setminus \bar{v}_a, f_v := \mathbf{true}, R := v \wedge \neg S$
- 5: **while** sat, with (sat, \mathbf{u}) := QSAT($\exists \bar{u}. \forall \bar{v}_a. \exists \bar{v}_e. R$) **do**
- 6: $\mathbf{u}_2 := \text{QCORE}(\mathbf{u}, \exists \bar{u}. \forall \bar{v}_a. \exists \bar{v}_e, \bar{x}'. \neg v \wedge \neg S)$
- 7: $f_v := f_v \wedge \neg \mathbf{u}_2, R := R \wedge \neg \mathbf{u}_2$
- 8: **DUMPCIRCUIT**(v, f_v), $S := S \wedge (v \leftrightarrow f_v)$

SYLEARNQBF builds up circuits in f_v for one $v \in \bar{v}$ after the other. Initially, $f_v = \mathbf{true}$, i.e., the circuit always outputs **true**. While there exists an input \mathbf{u} for which v must be **false** (the QBF in line 5 is satisfiable), f_v is refined with a clause that maps \mathbf{u} to **false**. By taking the core in line 6, other inputs are also mapped to **false** as long as **false** is allowed by S . The final solution f_v is dumped, and S is refined with the implementation of v before the next circuit is computed. The final f_v are in CNF, so the circuits have a depth of only two. Even after optimizations and mapping to standard cells, the depth usually remains low [7], which enables fast clock rates.

Once $\neg S$ is available in CNF, the algorithm only adds clauses to existing CNFs (i.e., to R and f_v). Only for the resubstitution in line 8, a CNF encoding of $\neg f_v$ is needed.

Optimizations for Safety Specifications. As in Sect. III-A, $\neg S$ is defined as $W \wedge T \wedge \neg W'$. This requires a CNF encoding of $\neg W'$. While computing $\neg W'$ with NEGLEARN is beneficial for QBF Cert, it does not pay off for SYLEARNQBF. Hence, we build a CNF for $\neg W'$ with one auxiliary variable per clause of W' . Recently, the QBF solver DepQBF was equipped with incremental solving capabilities [11]. SYLEARNQBF is well suited for incremental solving. We use two solver instances for line 5 and 6 respectively. For each $v \in \bar{v}$, a new incremental session is started. During the inner loop, we only add clauses to the former solver. The QBF of the latter even stays the same. DepQBF supports unsatisfiable cores natively. The resulting cores are small but not necessarily minimal, so we iterate over the remaining literals to obtain even smaller cores because (slightly) smaller cores typically mean (much) less iterations.

C. Interpolation

Jiang et al. [8] present two interpolation-based approaches to synthesize circuits for one $v \in \bar{v}$ after the other. The first one

expands S over \bar{v} . We consider this intractable in our setting. The second approach circumvents the quantifier alternation and expansion by temporarily treating output signals as inputs:

```

1: procedure SYINT( $S(\bar{x}, \bar{i}, \bar{o}, \bar{x}')$ )
2:    $\bar{d} := \bar{x} \cup \bar{i} \cup \bar{o} \cup \bar{x}'$ ,  $\bar{r} := \emptyset$ 
3:   for  $v \in \bar{v}$  do
4:      $\bar{d} := \bar{d} \setminus \{v\}$ ,  $\bar{r} := \bar{r} \cup \{v\}$ 
5:      $\bar{r}_1, \bar{r}_2, \bar{r}_3, \bar{r}_4 := \text{create4FreshCopies}(\bar{r})$ 
6:      $M_1(\bar{d}, \bar{r}_1, \bar{r}_2) := (S \wedge v)[\bar{r} \leftarrow \bar{r}_1] \wedge (\neg S \wedge \neg v)[\bar{r} \leftarrow \bar{r}_2]$ 
7:      $M_0(\bar{d}, \bar{r}_3, \bar{r}_4) := (S \wedge \neg v)[\bar{r} \leftarrow \bar{r}_3] \wedge (\neg S \wedge v)[\bar{r} \leftarrow \bar{r}_4]$ 
8:      $f_v(\bar{d}) := \text{INT}(M_1(\bar{d}, \bar{r}_1, \bar{r}_2), M_0(\bar{d}, \bar{r}_3, \bar{r}_4))$ 
9:      $\text{DUMPCIRCUIT}(v, f_v)$ ,  $S := S \wedge (v \leftrightarrow f_v)$ 

```

In each iteration, \bar{d} contains all variables on which the implementation of the current $v \in \bar{v}$ can depend, and \bar{r} contains the rest. For $\bar{v} = (v_1, \dots, v_n)$, v_1 can depend not only on \bar{u} but also on (v_2, \dots, v_n) , v_2 can depend on \bar{u} and (v_3, \dots, v_n) , etc. Yet, when the circuits for all $v \in \bar{v}$ are built together, the signals \bar{v} effectively depend on \bar{u} only. The formulas M_1 and M_0 characterize the \bar{d} -vectors for which v must be set to **true** and **false** respectively. The syntax $[\bar{r} \leftarrow \bar{r}_i]$ means that the variables \bar{r} are renamed by fresh copies \bar{r}_i . Line 8 computes an interpolant between M_1 and M_0 . The property $M_1 \Rightarrow f_v \Rightarrow \neg M_0$ of the interpolant ensures that (a) f_v is **true** whenever it must be **true**, and (b) whenever f_v is **true** then it does not have to be **false**. The renaming of the variables \bar{r} has the effect that f_v can only depend on the shared signals \bar{d} .

Optimizations for Safety Specifications. In order to avoid double-negations of W in S by negating S , we compute

$$M_1 := (T \wedge W' \wedge v)[\bar{r} \leftarrow \bar{r}_1] \wedge (T \wedge \neg v \wedge W \wedge \neg W')[\bar{r} \leftarrow \bar{r}_2]$$

$$M_0 := (T \wedge W' \wedge \neg v)[\bar{r} \leftarrow \bar{r}_3] \wedge (T \wedge v \wedge W \wedge \neg W')[\bar{r} \leftarrow \bar{r}_4]$$

Note the difference to a plain substitution of $S = T \wedge (\neg W \vee W')$ and $\neg S = T \wedge W \wedge \neg W'$ in SYINT: $(\neg W \vee W')$ reduces to W' due to the conjunction with W from $\neg S$. This is fortunate because disjunctions are expensive in CNF. Since SYINT allows v_i to depend on other v_j with $j > i$, it is sensitive to the variable order, both regarding execution time and circuit size. We exploit this insight with the following optimization. Once v_i has been synthesized, we analyze on which v_j it actually depends. If v_i does not depend on a particular v_j , then v_j is allowed to depend on v_i . This gives an increased flexibility without introducing circular dependencies. We simplify all computed interpolants with ABC⁵ [5].

D. SAT-Based Learning

Here, we use SYINT but with a special interpolation procedure (called in line 8) that applies computational learning:

```

1: procedure INTLEARN( $M_1(\bar{d}, \bar{r}_1, \bar{r}_2)$ ,  $M_0(\bar{d}, \bar{r}_3, \bar{r}_4)$ )
2:    $f := \text{true}$ 
3:   while sat, with (sat,  $\mathbf{d}$ ) := PSAT( $M_0 \wedge f$ ) do
4:      $f := f \wedge \neg \text{PCORE}(\mathbf{d}, M_1)$ 
5:   return  $f$ 

```

⁵We use the command sequence `strash; refactor -z1; rewrite -z1; up to 3 times, followed by dfraig; rewrite -z1; dfraig;`

As long as there exists some \mathbf{d} for which f is **true** but must be **false**, i.e., $M_0 \wedge f$ is satisfiable, we refine f with an additional clause that excludes the cube \mathbf{d} witnessing this insufficiency. By taking the unsatisfiable core, other inputs are also mapped to **false** as long as **false** is allowed.

Optimizations. We use two SAT solver instances, one for line 3 and one for line 4. A new incremental session is started upon each call of INTLEARN. Using activation variables to set \bar{v} -variables to **true**, **false**, or equal to their renamed copy, we can even use one incremental session throughout the entire SYINT procedure. However, this did not result in significant improvements in our experiments. All optimizations discussed in Sect. III-C can be applied. We also extended the variable dependency optimization further: The CNF T often contains many auxiliary variables that are defined uniquely by other signals of $\bar{x}, \bar{i}, \bar{o}$. If some of these auxiliary variables depend only on \bar{d} , then we allow f to depend on them as well by including them into \bar{d} . This can be beneficial because these auxiliary variables often capture the important connections between the variables $\bar{x}, \bar{i}, \bar{o}$. When dumping the circuits, we add additional gates that define the referenced auxiliary variables as done by T . We also implemented a second minimization pass that tries to remove every clause and literal from every CNF f after SYINT is done. However, this only results in minor circuit size improvements (around 20%).

IV. EXPERIMENTAL RESULTS

A. Implementation

We implemented the discussed methods and optimizations in the SAT-based synthesis tool Demiurge⁶ [4]. Demiurge synthesizes AIGER⁷ circuits from safety specifications and complies with the SyntComp⁸ competition rules. The archive of version 1.1.0 contains way more experiments than reported here. E.g., for the SAT-based learning approach alone we implemented 24 variants. Here, we only compare interesting versions, summarized in the following table.

Name	Engine	Algorithm
BDD	CuDD 2.4.2	Cofactor-Based [3]
QC	QBF Cert 1.0	QBF-Certification (Sect III-A)
QL	DepQBF 3.02	SYLEARNQBF (Sect III-B)
SI	MathSAT 5	SYINT (Sect III-C)
SL	Lingeling ats	SYINT+INTLEARN (Sect III-D)
SLN	Lingeling ats	SL without dependency opt.

BDD serves as baseline for our comparison. It was created by students and won a competition held in a synthesis lecture. It implements a cofactor-based approach [3], uses dynamic variable reordering, and forced reorderings at certain points. QC, QL, SI, and SL implement the methods from the previous section with all optimizations. SLN is used to highlight the benefits of the dependency optimization. All our methods use ABC⁵ [5] to minimize the final circuits further. SI uses

⁶http://www.iaik.tugraz.at/content/research/design_verification/demiurge/.

⁷<http://fmv.jku.at/aiger/>

⁸<http://www.syntcomp.org/>

MathSAT, which supports several interpolation schemes. We use McMillan’s scheme (see [6]), but the performance is similar with other schemes. We also implemented our own interpolation engine by processing proofs produced by PicoSat. However, the proof files grew prohibitively large.

The limitations of our implementation are that it can only handle safety specifications in AIGER format, it can produce circuit only in AIGER format, and it runs under Linux only.

B. Benchmarks

We use the same benchmarks as [4], but report here only results for the interesting ones. The benchmarks *amba_ij* specify an arbiter for ARM’s AMBA AHB bus [3], where *i* is the number of masters, and $j \in \{c, b, f\}$ indicates the method used to transform the original benchmark [3] into our input format [4]. The benchmarks *genbuf_ij*, again with $j \in \{c, b, f\}$, define a generalized buffer [3] connecting *i* senders to two receivers. The specifications *add_i* and *mult_i* denote *i*-bit combinational adders and multipliers.

C. Results and Discussion

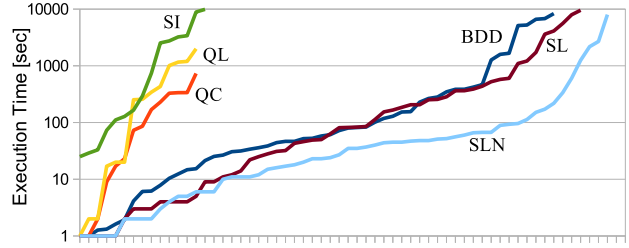
Fig. 2 summarizes our results with cactus plots. The y-axis gives the execution time or circuit size, and the x-axis gives the number of benchmarks that can be solved within this time or size limit. Concrete numbers and more plots can be found in the appendix and in the downloadable archive. All experiments were performed on an Intel Xeon E5430 CPU running a 64 bit Linux at 2.66 GHz. The memory limit was set to 8 GB, the time-out to 10 000 seconds. All circuits have been successfully model checked.

Method SL achieves the best results both regarding execution time and circuit size. The dependency optimization (SL vs. SLN) is very beneficial for *add* and *mult*, but slower for *amba* and *genbuf*. QC, QL, and SI do not perform so well. Using incremental QBF solving in QL gives an average speedup of factor 3.5. The speedup factor compared to using the QBF preprocessor Bloqqer is even 21. Still, QL is not very competitive. BDD is much better, but still outperformed by SL. In particular, SL outperforms SI by many orders of magnitude. Hence, our idea of implementing the interpolation procedure with computational learning is very beneficial. Execution time and circuit size are not in conflict but rather correlate. The time for optimization with ABC is usually insignificant, but only yields moderate size reductions (around 25 % for SL). Using method SLN, Demiurge won a track of SyntComp 2014. One reason was the small circuit size compared to other tools.

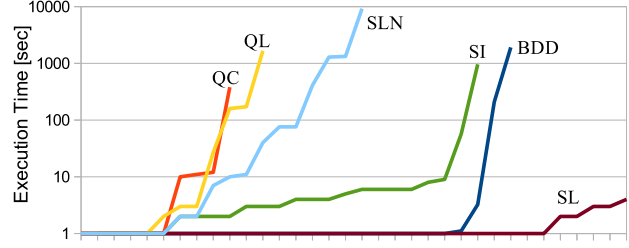
V. CONCLUSION

We compared several SAT- and QBF-based methods to synthesize circuits from strategies, and presented optimizations and efficient implementations for safety specifications. Our SAT-based learning method combines the quantifier elimination approach by Jiang et al. [8] with computational learning as proposed by Ehlers et al. [7], and outperforms BDDs both regarding execution time and circuit size in our experiments.

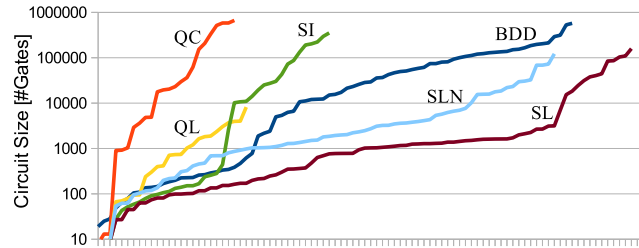
Future research includes preprocessing for incremental QBF solving, exploiting unreachable states, and parallelization.



(a) Execution time for *amba* and *genbuf*.



(b) Execution time for *add* and *mult*.



(c) Circuit size for all benchmarks.

Fig. 2. Cactus plots summarizing our performance evaluation.

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APPENDIX

Table I contains more performance results. “T” indicates a time-out of 10 000 seconds. The suffix k stands for a multiplication with 1000. The size column G gives the number of ALGER gates defining T . The column “files” in QC gives the size of the intermediate files (the QBF trace) produced by QBF Cert; we aborted at 20 GB. “M” indicates that ABC ran out of memory (because QC produces huge circuits). More details can be found in the downloadable archive.

TABLE I
PERFORMANCE RESULTS.

	Size				BDD		QC			QL		SI		SL		SLN	
	$ \bar{i} $	$ \bar{c} $	$ \bar{x} $	G	time	size	time	size	files	time	size	time	size	time	size	time	size
	[-]	[-]	[-]	[-]	[sec]	[cells]	[sec]	[cells]	[MB]	[sec]	[cells]	[sec]	[cells]	[sec]	[cells]	[sec]	[cells]
add2	4	2	2	17	0.1	2	1	13	0.1	0.1	9	1	9	1	9	0.1	9
add4	8	4	2	45	0.1	4	1	919	0.3	0.1	94	1	28	1	27	0.1	104
add6	12	6	2	73	0.1	6	11	20k	11	3.0	739	1	43	1	45	2	688
add8	16	8	2	101	0.1	8	164	M	738	172	4.0k	2	59	1	63	10	3.6k
add10	20	10	2	129	0.1	10	T	-	>17k	T	-	2	79	1	81	76	16k
add12	24	12	2	157	0.1	12	-	-	>20k	T	-	3	97	1	99	1.3k	69k
add14	28	14	2	185	0.1	14	-	-	>20k	T	-	4	113	1	117	T	-
add16	32	16	2	213	0.1	16	-	-	>20k	T	-	4	132	1	135	T	-
add18	36	18	2	241	0.1	18	-	-	>20k	T	-	6	151	1	153	T	-
add20	40	20	2	269	0.2	20	-	-	>20k	T	-	6	167	1	171	T	-
mult2	4	4	0	24	0.1	4	1	8	0.1	0.1	8	1	8	1	8	0.1	8
mult4	8	8	0	128	0.1	8	1	1.0k	6.1	2	414	3	440	1	95	1	412
mult5	10	10	0	217	0.2	10	12	4.9k	142	27	1.9k	9	2.6k	1	163	7	1.8k
mult6	12	12	0	322	1.1	12	380	23k	2.9k	1.7k	8.2k	57	14k	1	247	40	7.7k
mult7	14	14	0	455	3.2	14	-	-	>20k	T	-	962	73k	1	351	411	30k
mult8	16	16	0	604	209	16	-	-	>20k	T	-	T	-	1	477	9.2k	122k
mult10	20	20	0	964	T	-	-	-	>20k	T	-	T	-	1	777	T	-
mult12	24	24	0	1379	T	-	-	-	>20k	T	-	T	-	2	1.2k	T	-
mult16	32	32	0	2450	T	-	-	-	>20k	T	-	T	-	4	2.1k	T	-
genbuf1c	5	6	21	134	0.5	1.7k	1	2.9k	2.4	2	95	29	10k	1	101	1	62
genbuf2c	6	7	24	169	1.6	4.8k	9	18k	24.3	20	239	73	25k	3	198	2	119
genbuf3c	7	9	27	202	6.1	11k	86	62k	166	257	709	3.2k	191k	4	301	3	224
genbuf4c	8	10	30	242	12	15k	329	208k	1.3k	2.0k	1.0k	9.9k	222k	14	362	4	326
genbuf5c	9	12	33	284	21	21k	1.4k	M	4.7k	T	-	T	-	43	779	11	455
genbuf6c	10	13	35	323	36	36k	-	-	>20k	T	-	T	-	63	781	17	694
genbuf8c	12	15	40	406	282	61k	-	-	>20k	T	-	T	-	437	1.0k	44	1.0k
genbuf10c	14	18	45	494	155	100k	-	-	>20k	T	-	T	-	206	1.6k	45	1.5k
genbuf12c	16	20	49	561	266	154k	-	-	>20k	T	-	T	-	166	1.4k	51	1.1k
genbuf16c	20	24	58	733	465	203k	-	-	>20k	T	-	T	-	389	1.5k	47	2.0k
genbuf1b	5	6	23	141	0.8	2.3k	1	3.6k	3.1	1	70	25	11k	1	102	1	62
genbuf2b	6	7	26	174	6.2	6.5k	17	30k	31.7	17	399	111	19k	1	213	1	116
genbuf3b	7	9	30	208	7.8	11k	73	154k	354	252	1.6k	2.5k	202k	3	262	2	217
genbuf4b	8	10	33	245	34	26k	733	666k	2.3k	1.2k	1.2k	3.4k	352k	2	353	2	307
genbuf5b	9	12	37	282	52	29k	T	-	16k	T	-	T	-	5	693	5	863
genbuf6b	10	13	40	322	27	23k	-	-	>20k	T	-	T	-	11	1.3k	11	2.2k
genbuf8b	12	15	46	395	61	51k	-	-	>20k	T	-	T	-	12	1.1k	6	804
genbuf10b	14	18	53	475	118	80k	-	-	>20k	T	-	T	-	25	775	16	1.9k
genbuf12b	16	20	59	547	31	74k	-	-	>20k	T	-	T	-	46	1.0k	27	2.0k
genbuf16b	20	24	71	687	44	107k	-	-	>20k	T	-	T	-	113	1.3k	113	18k
genbuf1f	5	6	23	138	1.3	2.0k	2	4.8k	2.9	2	67	33	11k	1	74	0.1	54
genbuf2f	6	7	26	168	1.9	6.1k	23	36k	33	20	301	128	27k	1	218	0.1	136
genbuf3f	7	9	30	200	82	12k	169	332k	673	432	1.8k	2.7k	133k	4	372	2	198
genbuf4f	8	10	33	235	15	17k	2.7k	M	6.0k	T	-	T	-	9	762	10	476
genbuf5f	9	12	37	272	47	46k	-	-	>20k	T	-	T	-	49	1.6k	18	1.0k
genbuf6f	10	13	40	309	53	42k	-	-	>20k	T	-	T	-	185	2.7k	67	1.9k
genbuf7f	11	14	43	344	386	112k	-	-	>20k	T	-	T	-	524	3.1k	172	2.5k
amba2c	7	8	28	177	1.3	5.2k	229	516k	985	343	2.3k	294	43k	4	1.2k	6	1.2k
amba3c	9	10	34	237	10	15k	-	-	>20k	T	-	8.9k	296k	22	2.3k	12	1.3k
amba4c	11	11	38	279	229	129k	-	-	>20k	T	-	T	-	206	18k	93	10k
amba5c	13	13	43	345	1.7k	166k	-	-	>20k	T	-	T	-	256	15k	35	3.2k
amba6c	15	14	47	395	5.1k	131k	-	-	>20k	T	-	T	-	576	24k	40	3.8k
amba7c	17	15	52	449	6.8k	138k	-	-	>20k	T	-	T	-	1.2k	38k	67	4.3k
amba9c	21	18	61	583	T	-	-	-	>20k	T	-	T	-	4.1k	86k	151	5.4k
amba10c	23	19	65	630	1.6k	212k	-	-	>20k	T	-	T	-	5.6k	110k	218	7.0k
amba2b	7	8	31	189	15	12k	337	584k	1.3k	1.2k	3.7k	739	87k	4	1.3k	6	1.1k
amba3b	9	10	36	231	83	74k	-	-	>20k	T	-	T	-	28	3.1k	11	1.4k
amba4b	11	11	42	286	8.3k	314k	-	-	>20k	T	-	T	-	602	31k	336	18k
amba5b	13	13	47	344	6.6k	574k	-	-	>20k	T	-	T	-	1.1k	41k	45	4.1k
amba6b	15	14	52	391	T	-	-	-	>20k	T	-	T	-	3.6k	85k	60	5.7k
amba7b	17	15	57	438	T	-	-	-	>20k	T	-	T	-	8.0k	159k	96	6.6k
amba9b	21	18	68	558	T	-	-	-	>20k	T	-	T	-	T	-	605	22k
amba10b	23	19	73	606	T	-	-	-	>20k	T	-	T	-	T	-	2.7k	72k
amba2f	7	8	31	181	4.1	12k	336	584k	1.4k	1.0k	3.0k	165	30k	3	1.3k	5	1.0k
amba3f	9	10	37	229	101	36k	T	-	>16k	T	-	T	-	83	7.0k	52	6.2k
amba4f	11	11	43	282	1.3k	293k	-	-	>20k	T	-	T	-	1.7k	44k	1.3k	33k
amba5f	13	13	49	346	T	-	-	-	>20k	T	-	T	-	9.5k	104k	2.2k	23k
amba6f	15	14	54	391	T	-	-	-	>20k	T	-	T	-	T	-	8.0k	31k

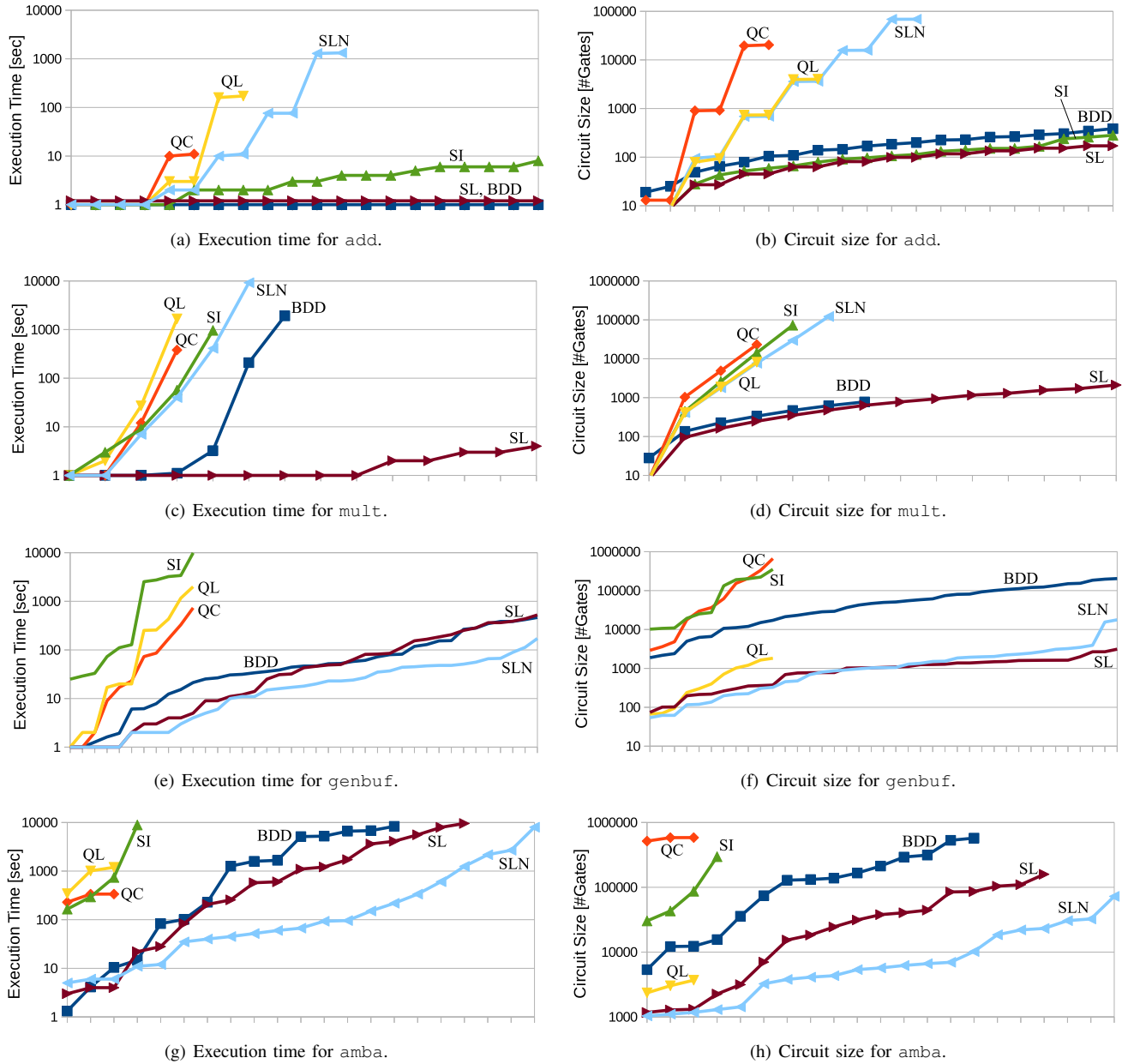


Fig. 3. Cactus plots summarizing execution time and circuit size per benchmark.

Fig. 3 contains cactus plots summarizing the execution time and circuit size per benchmark. For `mult`, the dependency optimization has an even stronger positive impact than for `add`, both regarding execution time and circuit size (SL vs. SLN). For `genbuf`, it results in smaller circuits, but is slower. For `amba`, it is not beneficial in either metric. The interpolation-based method SI is competitive for `add`, but has troubles with the other benchmarks. The difference in circuit size can grow very large (more than three orders of magnitude). Our a-posteriori circuit minimization using ABC can only compensate an insignificant fraction of this difference. Investing more effort in post-processing can be expensive, especially for large circuits. We therefore conclude

that circuit size is best considered during the synthesis process already. The fact that circuit size and execution time correlate is not surprising. Most of our methods compute circuits for one output signal after the other. The strategy is refined with the circuit for one signal before continuing with the next one. This is necessary in order to prevent uncoordinated choices. If the computed circuits are complicated, then this re-substitution makes the strategy formula for the next output complicated, which results in higher computation times. This may also explain why implementing an interpolation procedure with computational learning is very beneficial (SL vs. SI): computational learning seems to find smaller circuits, and this also pays off in terms of the overall computation time.