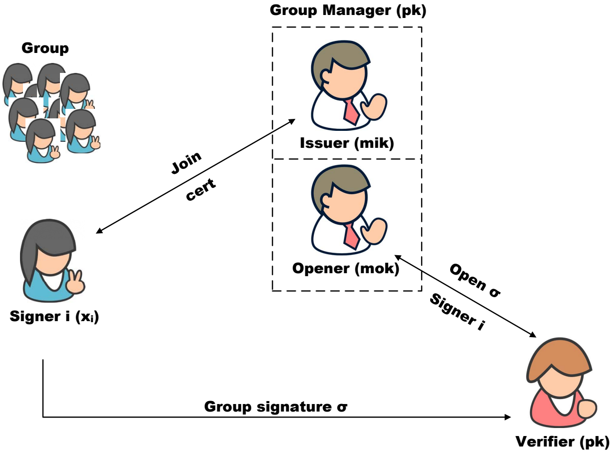
Non-Interactive Plaintext (In-)Equality Proofs and Group Signatures with Verifiable Controllable Linkability

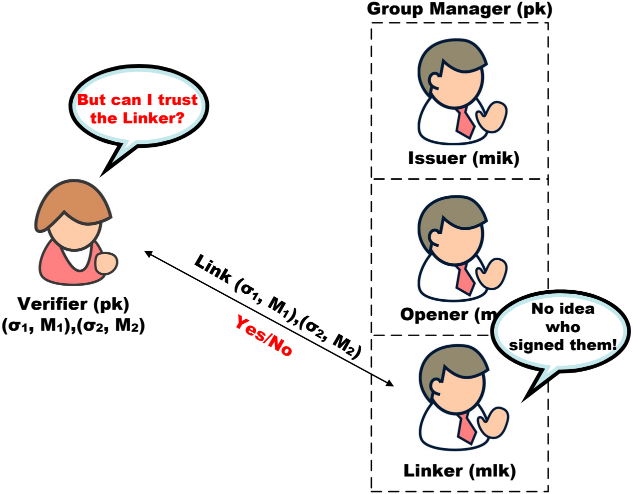
Olivier Blazy[1], David Derler[2], Daniel Slamanig[2], Raphael Spreitzer[2]
[1] **Université de Limoges, XLim, France**
[2] **IAIK, Graz University of Technology, Austria**

CT-RSA 2016, San Francisco, 2nd March 2016

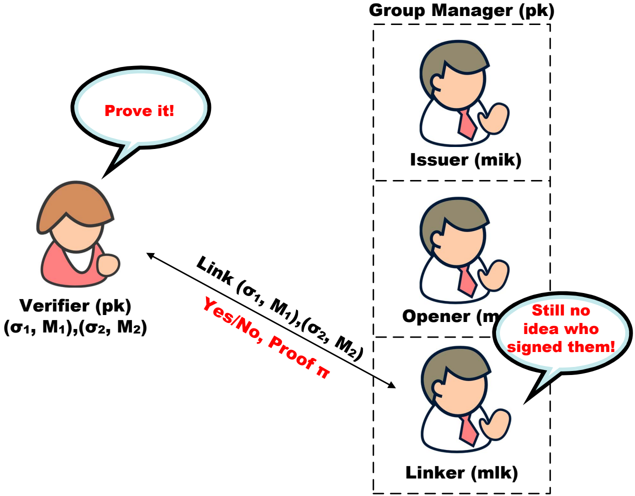
Group Signature Schemes [CvH91]



Controllable Linkability [HLhC^{+}11, SSU14]



Verifiable Controllable Linkability



Sign-Encrypt-Prove Paradigm

Basic building blocks

- $\mathcal{DS} = (\mathsf{KG}_\mathsf{s}, \mathsf{Sign}, \mathsf{Verify})$
- $\mathcal{AE} = (\mathsf{KG}_\mathsf{e}, \mathsf{Enc}, \mathsf{Dec})$
- Signature of Knowledge

Keys

- $gpk \leftarrow (pk_e, pk_s)$, $gmsk \leftarrow sk_e$, $gmik \leftarrow sk_s$

Join

- User's secret: x_i
- Issuer computes: $cert \leftarrow \mathsf{Sign}(gmik, f(x_i))$

Sign-Encrypt-Prove Paradigm I

Sign

- $T \leftarrow \mathrm{Enc}(pk_e, cert)$
- $\pi \leftarrow SoK\{(x_i, cert) : cert = \mathrm{Sign}(sk_s, f(x_i)) \wedge T = \mathrm{Enc}(pk_e, cert))\}(m)$
- $\sigma \leftarrow (T, \pi)$

Verify

- “verification of π”

Open

- $cert \leftarrow \mathrm{Dec}(sk_e, T)$

Contributions

1. Generic proof system for plaintext (in-)equality
2. Efficient instantiation of this proof system
3. Group signatures with verifiable controllable linkability
4. Extend GSs with verifiable controllable linkability (VCL)

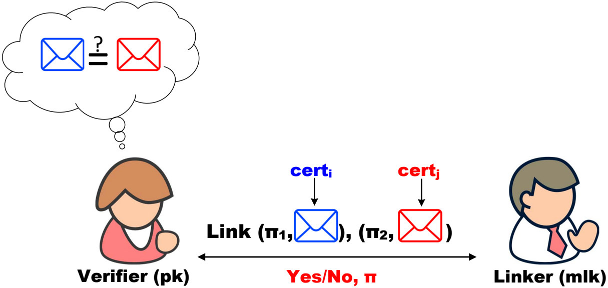
Controllable Linkability

Public key encryption with equality tests [Tan12, SSU14]

- Conventional public key encryption scheme
+ Com algorithm for equality tests using trapdoor
- $\Rightarrow$ Link: $1/0 \leftarrow \mathrm{Com}(T, T', gmlk)$

- Semantic security without trapdoor
- One-way security for trapdoor holders

Setting



Non-interactive plaintext (in-)equality proofs

Non-Interactive Plaintext (In-)Equality Proofs

Given any $\mathcal{PKEQ}$ and ciphertexts T and T' under pk

Proof system Π

1. Prove knowledge of trapdoor tk
2. $\mathsf{Com} = 1$ (membership) or $\mathsf{Com} = 0$ (non-membership)
3. Without revealing trapdoor tk

(Non-)Membership Proofs

Com $= 1$ defines language $L_{\in}$ for membership

- Witnessed by trapdoor tk
- Standard techniques [GS08]

Com $= 0$ defines language $L_{\notin}$ for non-membership

- Idea [BCV15]
  - Π_1: Failing membership proof for $L_{\in}$
  - Π_2: Proof that Π_1 has been computed honestly
- Efficient instantiations (GS and SPHFs)
- Technicalities: m, r must be known [BCV15]

Smooth Projective Hash Functions (SPHFs)

$w : (x, w) \in R$

Hashing key: hk
Projection key: hp

Verifier

Statement: $x \in L_R$

Prover

hp →

$H \leftarrow \mathit{ProjHash}(hp, x, w)$

← H

$H \overset{?}{=} \mathit{Hash}(hk, x)$

If $x \in L_R : \mathit{Hash}(hk, x) \overset{!}{=} \mathit{ProjHash}(hp, x, w)$
(Correctness)

Construction - Non-Membership Proof

Hashing key: hk
Projection key: hp

Verifier

Statement: $x \in L_R$

Prover

$H' \leftarrow Hash(hk, x)$

$\pi_2 \leftarrow Proof((H' \wedge hp), hk)$

$H \leftarrow ProjHash(hp, x, w)$

$\pi_1 = Proof((hp, x, H), w)$

$\xleftarrow{H, H', hp, \pi_1, \pi_2}$

$$H \stackrel{?}{\neq} H' \;\wedge\; Verify((hp, x, H), \pi_1) \stackrel{?}{=} 1 \;\wedge\; Verify((H' \wedge hp), \pi_2) \stackrel{?}{=} 1$$

Example of Efficient Instantiation

ElGamal with equality tests (as in [SSU14])

- Keypair: $(sk, pk) \leftarrow (x, g^{x}) \in \mathbb{Z}_p \times \mathbb{G}_1$
- Trapdoor: $(\hat{r}, \hat{r}^{x}) \in \mathbb{G}_2 \times \mathbb{G}_2$
- Encryption of m: $(g^{r}, m \cdot g^{x \cdot r}) \in \mathbb{G}_1 \times \mathbb{G}_1$

Pairing-based equality test

- Ciphertexts: $(g^{r}, m \cdot g^{x \cdot r}), (g^{r'}, m' \cdot g^{x \cdot r'})$

$$m = m' \iff \frac{e(m \cdot g^{x \cdot r}, \hat{r})}{e(g^{r}, \hat{r}^{x})} = \frac{e(m' \cdot g^{x \cdot r'}, \hat{r})}{e(g^{r'}, \hat{r}^{x})}$$

Instantiation of $\Pi_\in$

Com $=$ 1: plaintext equality proof

$$((g^{r}, m \cdot g^{x \cdot r}), (g^{r'}, m' \cdot g^{x \cdot r'}), g^{x}) \in L_\in \iff$$

$$\frac{e(m \cdot g^{x \cdot r}, \hat{r})}{e(g^{r}, \hat{r}^{x})} = \frac{e(m' \cdot g^{x \cdot r'}, \hat{r})}{e(g^{r'}, \hat{r}^{x})} \quad \wedge$$

$$e(g, \hat{r}^{x}) = e(g^{x}, \hat{r})$$

$$\prod_{i=1}^{2} e(A_i, \underline{\hat{Y}_i}) \;=\; \frac{e(m \cdot g^{x \cdot r} \cdot (m' \cdot g^{x \cdot r'})^{-1}, \hat{r})}{e(g^{r} \cdot g^{-r'}, \hat{r}^{x})} \;=\; 1_{\mathbb{G}_T}$$

Instantiation of $\Pi_{\notin}$

Com = 0: plaintext inequality proof

$$((g^{r}, m \cdot g^{x \cdot r}), (g^{r'}, m' \cdot g^{x \cdot r'}), g^{x}) \in L_{\notin} \iff$$

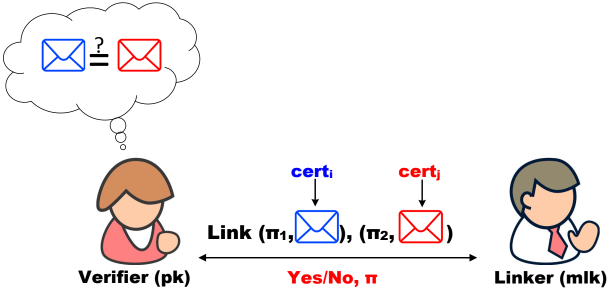
$$\frac{e(m \cdot g^{x \cdot r}, \hat{r})}{e(g^{r}, \hat{r}^{x})} \neq \frac{e(m' \cdot g^{x \cdot r'}, \hat{r})}{e(g^{r'}, \hat{r}^{x})} \quad \wedge$$

$$e(g, \hat{r}^{x}) = e(g^{x}, \hat{r})$$

$\Rightarrow$ Our construction for non-membership proofs

NIPEI Proof System

Proof system $\Pi = (\Pi_{\in}, \Pi_{\notin})$



GSSs with Verifiable Controllable Linkability

Extended security model for VCL-GS

- Algorithms: Link and $\text{Link}_{\text{Judge}}$
- Property: linking soundness

Instantiation based on NIPEI

- Link: Π.Proof
- $\text{Link}_{\text{Judge}}$: Π.Verify

Take-Home Message

- Proposed generic approach for (in-)equality proof
- Efficient instantiation in the pairing setting
- Rather independent of encryption scheme
  - Various DDH/DLIN ElGamal variants
  - CCA2: Naor-Yung and Cramer-Shoup (for free)
- Novel application
  - GSSs with verifiable controllable linkability

Non-Interactive Plaintext (In-)Equality Proofs and Group Signatures with Verifiable Controllable Linkability

Olivier Blazy[1], **David Derler**[2], **Daniel Slamanig**[2], **Raphael Spreitzer**[2]
[1] **Université de Limoges, XLim, France**
[2] **IAIK, Graz University of Technology, Austria**

CT-RSA 2016, San Francisco, 2nd March 2016

Bibliography I

[BCV15] Olivier Blazy, Céline Chevalier, and Damien Vergnaud. Non-Interactive Zero-Knowledge Proofs of Non-Membership. In *CT-RSA*, 2015.

[CvH91] David Chaum and Eugène van Heyst. Group Signatures. In *EUROCRYPT*, 1991.

[GS08] Jens Groth and Amit Sahai. Efficient Non-interactive Proof Systems for Bilinear Groups. In *EUROCRYPT*, 2008.

[HLhC+11] Jung Yeon Hwang, Sokjoon Lee, Byung ho Chung, Hyun Sook Cho, and DaeHun Nyang. Short Group Signatures with Controllable Linkability. In *LightSec*. IEEE, 2011.

[SSU14] Daniel Slamanig, Raphael Spreitzer, and Thomas Unterluggauer. Adding Controllable Linkability to Pairing-Based Group Signatures for Free. In *ISC*, 2014.

[Tan12] Qiang Tang. Public Key Encryption Supporting Plaintext Equality Test and User-Specified Authorization. *Security and Communication Networks*, 5(12), 2012.